Consumer Discrete Choice Models

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November 24, 2014

Introduction

- Analysis of the choices made by agents among a finite and discrete set of alternatives
- DCM help us to predict the probability that an agent chooses a particular alternative (imperfect information)
- Applications: Marketing, Online Behavior, Transportation Planners, Regulators,...

Introduction

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- Applications: Marketing, Online Behavior, Transportation Planners, Regulators,...
- References:
  - Kenneth Train (2009): "Discrete Choice Methods with Simulation". Cambridge University Press
  - Michael Keane (2013): "Panel Data Discrete Choice Models of Consumer Demand"
  - Berry, Levinsohn, and Pakes (1995):"Automobile Prices in Market Equilibrium"
  - Nevo (2001):"Measuring Market Power in the Ready-to-Eat Cereal Industry"

Consumer Utility

- Choice Set: alternatives must be mutually exclusive, exhaustive, finite.
- Let $j = 1, ..., J$ alternatives and $i = 1, ..., I$ individuals.

$$U_{ij} = \beta_i X_{ij} + \epsilon_{ij}$$

$$y_{ij} = \begin{cases} 1 & if \quad U_{ij} > U_{ik} \quad \forall j \neq k \\ 0 & otherwise \end{cases}$$

- U_{ij}: latent utility that individual i obtains from alternative j
- y_{ij}: observed choice
- X_{ij}: observed covariates (product and demographic characteristics)
- β_i: vector of parameters linked to covariates
- ϵ_{ij}: unobserved factors
- Example: Car Industry (Sport Car vs Van)

$$U_{ij} = f(Price_j, HP_j, Size_j, AC_j, Miles\$_j, Ads_j, Inc_i, FamilySize_i) + \epsilon_{ij}$$

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- Identification of choice models: only differences in utility matter, the scale of utility is arbitrary.

Choice Probability

$$
\begin{aligned}
P_{ij} &= Prob(y_{ij} = 1) = Prob(U_{ij} > U_{ik} \;\; \forall j \neq k) \\
&= Prob(\beta_i X_{ij} + \epsilon_{ij} > \beta_i X_{ik} + \epsilon_{ik} \;\; \forall j \neq k) \\
&= \int \mathbb{1}(\epsilon_{ik} - \epsilon_{ij} < \beta_i X_{ij} - \beta_i X_{ik} \;\; \forall j \neq k) f(\epsilon_i) d\epsilon_i
\end{aligned}
$$

where $\epsilon_i = (\epsilon_{i1}, ..., \epsilon_{iJ})$

- ϵ, β structure: logit, nested logit, probit, mixed-logit, mixed-probit models

Choice Probability

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where $\epsilon_i = (\epsilon_{i1}, ..., \epsilon_{iJ})$

- ϵ, β structure: logit, nested logit, probit, mixed-logit, mixed-probit models
- Logit model: $\epsilon_i \sim iidEV1$

$$P_{ij} = \frac{e^{\beta_i X_{ij}}}{\sum_k e^{\beta_i X_{ik}}}$$

Strong Assumption: Independence from Irrelevant Alternatives.

Choice Probability

$$
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\end{aligned}
$$

where $\epsilon_i = (\epsilon_{i1}, ..., \epsilon_{iJ})$

- Probit model: $\epsilon \sim \mathcal{N}(0, \Omega)$

$$
f(\epsilon_i) = \frac{1}{(2\pi)^{J/2}|\Omega|^{1/2}} exp\{ -\frac{1}{2}\epsilon_i'\Omega^{-1}\epsilon_i\}
$$

Choice Probability

$$
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P_{ij} &= Prob(y_{ij} = 1) \\
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&= Prob(\beta_i X_{ij} + \epsilon_{ij} > \beta_i X_{ik} + \epsilon_{ik} \;\; \forall j \neq k)
\end{aligned}
$$

where $\epsilon_i = (\epsilon_{i1}, ..., \epsilon_{iJ})$

- Mixed-logit model: $\beta_i \sim \mathcal{N}, LogNormal, ...$ and $\epsilon \sim EV1$

$$
P_{ij} = \int \frac{e^{\beta_i X_{ij}}}{\sum_k e^{\beta_i X_{ik}}} f(\beta) d\beta
$$

- Example: Car Industry (Sport Car vs Van)

$$
U_{ij} = \alpha_0 + (\beta_{10} + \beta_{11} Income_i + \beta_{12}\nu_i) Price_j + (\beta_{20} + \beta_{21} FamilySize_i) Size_j + \epsilon_{ij}
$$

$$
FamilySize_i \sim DistA \;\; , Income_i \sim DistB \;\; , \;\; \nu_i \sim ln\mathcal{N}(0, 1)
$$

Elasticities and cross-elasticities

- Elasticities

$$E_{jx_{ik}^m} = \frac{\partial P_{ij}}{\partial x_{ik}^m} \frac{x_{ik}^m}{P_{ij}}$$

- Logistic Model:

$$E_{jx_{ik}^m} = \begin{cases} \beta^m x_{ij}^m (1 - P_{ij}) & if \quad k = j \\ \\ -\beta^m x_{ik}^m P_{ik} & if \quad k \neq j \end{cases}$$

- Mixed-logit Model:

$$E_{jx_{ik}^m} = -\frac{x_{ik}^m}{P_{ij}} \int \beta^m L_{ij}(\beta) L_{ik}(\beta) f(\beta) d\beta$$

$$L_{ij} = \frac{e^{\beta_i X_{ij}}}{\sum_u e^{\beta_i X_{iu}}}$$

Elasticities and cross-elasticities

Berry, Levinhson, and Pakes (1995)

- Develops a new set of tools to analyze demand and supply in differentiated products markets (Mixed-Logit)
- Application in the U.S. automobile industry

Table: Own and Cross-Price Semi-Elasticities

	Mazda 323	Ford Scort	Lexus LS400	BMW 735i
Mazda 323	-25	9.954	0.002	0.000
Ford Escort	0.713	-20	0.003	0.000
Lexus LS400	0.001	0.018	-11	0.086
BMW 735i	0.000	0.009	0.336	-9

Estimation

- Sample of I independent decision makers
- Likelihood

$$L(\beta) = \Pi_{i=1}^{I} \Pi_j (P_{ij})^{y_{ij}}$$

- Maximum Likelihood Estimator

$$\hat{\beta} = argmax_{\beta} \sum_{i=1}^{I} \sum_{j} y_{ij} ln P_{ij}$$

Estimation

Nevo (2001)

- Test if the ready-to-eat cereal industry colludes on prices
- Conclusion: prices in the industry are consistent with noncollusive pricing behavior

$$
\begin{aligned}
U_{ij} &= \alpha + (\beta_1 + \sigma_{11} Inc_i + \sigma_{12}\nu_i^{(1)}) Price_j + \beta_2 Adv_j \\
&+ (\beta_3 + \sigma_{31} Inc_i + \sigma_{32} Age_i + \sigma_{33}\nu_i^{(3)}) Sugar_j + ... + \epsilon_{ij}
\end{aligned}
$$

where

$$
Age_i \sim DistA \ , Income_i \sim DistB \ , \ \ \nu_i^{(s)} \sim \mathcal{N}(0,1)
$$

Table: Demand Estimates

	Means (β)	Deviations (σ)	Income	Age
Price	-27	2.453	316	–
	(5.248)	(2.978)	(110)	–
Advertising	0.020	–	–	–
	(0.005)	–	–	–
Sugar	5.742	1.661	-24.931	5.105
	(0.581)	(5.866)	(9.167)	(3.418)
...	...	...	...	...

Panel Data

- Let $j = 1, ..., J$ index alternatives, $t = 1, ..., T$ index time, and $i = 1, ..., I$ index people

$$U_{ijt} = \alpha_{ij} + X_{ijt}\beta + \gamma y_{ij,t-1} + \epsilon_{ijt}$$

where

$$\epsilon_{ijt} = \rho\epsilon_{ij,t-1} + \eta_{ijt}$$

$$y_{ijt} = \begin{cases} 1 & if \quad U_{ijt} > U_{ikt} \quad \forall j \neq k \\ 0 & otherwise \end{cases}$$

- X_{it} vector of product and consumer attributes
- β attribute weights
- α_{ij} consumer i's intrinsic preference for brand j
- $y_{ij,t-1}$ choice at t-1
- γ weight of the previous decision
- ϵ_{it} person, time and brand specific taste shock
- ρ temporal persistence shock
- η_{ijt} iid fundamental shock

Panel Data

- Assuming no correlated taste shocks ($\rho = 0$)
- Logit Model

$$P_{ij} = \Pi_{t=1}^{T} P_{ijt} = \Pi_{t=1}^{T} \left[\frac{e^{\beta X_{ij_t t}}}{\sum_k e^{\beta X_{ikt}}} \right]$$

- Mixed Logit Model

$$P_{ij} = \int \Pi_{t=1}^{T} \left[\frac{e^{\beta_i X_{ij_t t}}}{\sum_k e^{\beta_i X_{ikt}}} \right] f(\beta) d\beta$$

- Likelihood

$$L(\beta) = \Pi_{i=1}^{I} \Pi_j (P_{ij})^{y_{ij}}$$