Estimation of a Dynamic Auction Game

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- **Estimation Method:** Two stage approach:
  1. The distribution of bids conditional on state variables is estimated using data on bids, bidder characteristics and contract characteristics
  2. An expression of the expected sum of future profits based on the distribution of bids is obtained, and costs are **inferred** based on the first order condition of optimal bids

Introduction:

- **Application:** Repeated procurement auctions for highway paving contracts.
- Previously won uncompleted contracts may affect the ability of win further contracts. Two effects:
  - Winning a large contract may commit some of the bidder's machines and paving resources for the duration of the contract. Increasing the costs for future contracts.
  - Expertise may lower the cost for future contracts.
- **Results:** Capacity constraints affect firms' bidding strategies. The cost of taking on an additional contract is increasing in backlog. The increase in costs resulting from a larger than average backlog seems to cancel out any cost-reducing expertise effects.

The data and industry

- Data from Caltrans contract awards for highway and street construction made between dec 1988 and May 1999
- During this periods 2207 projects were awarded
- The Bid data contains the following info: bid opening date; contract number;location;reservation price;number of working days and the engineers' estimate; name, address, amount of the bid, and the rank of the bid for each of the bidding firms
- Bidders may submit a sealed bid and they do not know who else submits a bid
- The project is awarded to the lowest bidder provided it is below the reserve price.
- The reserve price consists of a fixed nonrandom $ amount (R^t) assigned prior to the bidding. Existence of a secret reserve price.

The data and industry

TABLE I
DESCRIPTIVE STATISTICS OF SELECTED VARIABLES

	Number of Observations	Mean	Standard Deviation	Minimum	Maximum
Number of Bidders	2223	4.63	2.46	0.00	19.00
Estimate[a]	2223	13.41	1.35	9.47	18.31
(Ranked1[b]−Estimate)/Estimate	2207	−0.04	0.22	−0.79	3.07
(Ranked2[b]−Ranked1)/Ranked1	2111	0.09	0.12	0.00	2.62
Backlog[c]	22230	0.00	1.00	−3.24	2.97

[a]Logarithm of the engineers' estimate.

[b]Ranked1 and Ranked2 are the winning bid and the bid ranked in second position, respectively.

[c]Backlog measures the $ value of previously won uncompleted contracts. It is standardized by subtracting the bidder specific mean and dividing by the bidder specific standard deviation.

- **Backlock**

The data and industry

TABLE II

DESCRIPTIVE STATISTICS OF SELECTED VARIABLES BY NUMBER OF BIDDERS

	Number of Bidders:	All	0	1	2	3	4	5	6	7–8	9–19
	# Observations:	2223	16	96	285	393	432	356	237	251	157
Estimate[a]											
	Mean	13.41		13.14	13.47	13.49	13.32	13.42	13.55	13.48	13.12
	Standard Deviation	1.35		1.04	1.27	1.33	1.29	1.35	1.59	1.47	1.19
(Ranked1[b]−Estimate)/Estimate											
	Mean	−0.04		0.11	0.03	−0.01	−0.04	−0.06	−0.09	−0.10	−0.14
	Standard Deviation	0.22		0.36	0.29	0.21	0.20	0.19	0.16	0.20	0.16
(Ranked2[b]−Ranked1)/Ranked1											
	Mean	0.09			0.14	0.11	0.09	0.08	0.06	0.07	0.06
	Standard Deviation	0.12			0.11	0.19	0.10	0.09	0.06	0.07	0.07

[a] Logarithm of the engineers' estimate.

[b] Ranked1 and Ranked2 are the winning bid and the bid ranked in second position, respectively.

- As the number of bidders increases, the relative difference between the low bid and the Caltrans estimate falls.
- Money Left on the table
- Fringe vs Regular Bidders: More than 500 bidders submit a bid at least once. Most of these bidders submit a bid once, or few occasions (Fringe). The number of regular bidders per contract ranges from 0 to 4 and has an average of 0.53. Regular bidders win 25% of the total dollar value awarded and 17% of all contracts.

Reduced Form Estimates

TABLE III
BID SUBMISSION AND BID LEVEL DECISIONS

Estimation Method:	Probit[a]			Tobit[a]			Heckman[a]		
Dependent Variable:	Bid Submission			(R−Bid)/Estimate[b]			(R−Bid)/Estimate[b]		
Number of Observations:	22230	22230	22230	22230	22230	22230	22230	22230	22230
Chi-square:	1605.65	1984.17	1677.77	1518.99	1883.42	1589.23	420.41	444.64	420.44
Degrees of Freedom:	6	15	15	6	15	15	6	15	15
Log Likelihood:	−4281.26	−4092.39	−4245.60	−3765.05	−3582.84	−3729.93	−3404.46	−3394.35	−3366.71
Variable									
Constant	−2.8485	−3.169	−2.864	−1.1654	−1.2511	−1.1644	0.3093	0.2734	0.3207
	(0.173)	(0.184)	(0.174)	(0.089)	(0.091)	(0.089)	(0.072)	(0.076)	(0.073)
Estimate	0.2905	0.3024	0.2902	0.1235	0.1220	0.1250	0.0040	0.0038	0.0032
	(0.015)	(0.016)	(0.015)	(0.008)	(0.008)	(0.008)	(0.006)	(0.007)	(0.007)
Working Days	−0.3176	−0.3234	−0.3270	−0.1498	−0.1446	−0.1537	−0.0533	−0.0540	−0.0527
	(0.022)	(0.023)	(0.023)	(0.011)	(0.011)	(0.011)	(0.008)	(0.009)	(0.008)
Nbid-Fringe	−0.1835	−0.1913	−0.1885	−0.0882	−0.0875	−0.0905	−0.0613	−0.0599	−0.0624
	(0.027)	(0.027)	(0.027)	(0.013)	(0.013)	(0.013)	(0.007)	(0.008)	(0.008)
Distance	−0.5193	−0.4805	−0.5238	−0.2536	−0.2240	−0.2543	−0.1196	−0.0978	−0.1188
	(0.023)	(0.024)	(0.023)	(0.012)	(0.012)	(0.012)	(0.008)	(0.009)	(0.009)
# Plants	0.1807	0.0513	0.1786	0.0638	0.0078	0.0632	−0.0051	−0.0193	−0.0052
within Region	(0.051)	(0.054)	(0.052)	(0.025)	(0.024)	(0.024)	(0.014)	(0.015)	(0.015)
Backlog	−0.0835	−0.0856	−0.1079	−0.0383	−0.0372	−0.0528	−0.0127	−0.0127	−0.0162
	(0.015)	(0.015)	(0.053)	(0.007)	(0.007)	(0.025)	(0.004)	(0.005)	(0.017)
Firm_2		0.6784			0.2985			0.1204	
		(0.061)			(0.029)			(0.019)	
Firm_3		−0.0338			−0.0081			−0.0223	
		(0.073)			(0.034)			(0.024)	

- A prediction is that the constrained bidders bid less frequently and higher than unconstrained bidders.
- The estimates support our dynamic bidding model in 3 ways: 1, capacity constraints appear important. 2, bidders additional state vars, location, and size are important. 3, a test of identical backlog effects cannot be rejected for the majority of bidders (symmetry)

Reduced Form vs Structural:

- Why do we need to introduce economic theory?
  - Interaction between players and variables (e.g. probability of winning depends on others' bids, bids depend on unobserved costs, equilibrium assumptions,...)
  - Counterfactuals (Policy Changes)

The Bidding Model:

- Discrete time with infinite horizon with two types of bidders
- Regular bidders stay forever and are denoted by $\{1, ..., n_r\}$ and fringe stays one period t and denoted by $\{n_r + 1, ..., n^t\}$
- The stage game: In period t the buyer offers a single contract for sale.
  1. The characteristics of the contract are revealed to all bidders.
  2. Bidders learn their costs privately.
  3. Bidders may submit bids.
  4. The buyer may award the contract to the low bidder or reject all bids.
- The contract characteristics s_0^t are drawn iid from the exogenous $F_0(.)$, and are assumed to be unknown to bidders.
- s_0^t include the physical attributes of the contract such as the contract size and duration, the total number of fringe bidders, and a fixed reserve price of the seller R^t which is an upper bound on admissible bids.
- **Costs:** each bidder i learns his cost for the contract c_i^t after the contract characteristics are revealed. The cost is privately known and iid conditional on state variables. The cost of a regular bidder i is drawn from the conditional distribution $F(.|s_0^t, s_i^t, s_{-i}^t)$.

The Bidding Model:

- Bidder i's state vector s_i records the remaining size in dollars and the number of days left until completion of each project previously won by bidder i.
- Bids: b. All agents are risk neutral
- The buyer awards the contract to the bidder with the low bid at a price equal to her bid.
- The buyer may decide to reject all bids if the lowest bid is greater than R^t. (Existence of a secret reserve price)

The Bidding Model:

- The transaction function of the state variable ω is a deterministic function of the contract characteristics and the state variables
- Let the *ith* component of the transition function ω consist of the list of the sizes and remaining times of all the project left to do for bidder i. It can be written as

$$\omega_i(s_0, s, j) = \begin{cases} ((z_0, \tau_0), (\frac{max(\tau_i^l - 1, 0)}{\tau_i^l} z_i^l, max(\tau_i^l - 1, 0))_{l=1}^{\bar{\tau}-1}) \textit{ if contract winner } j = i \\ ((0,0), (\frac{max(\tau_i^l - 1, 0)}{\tau_i^l} z_i^l, max(\tau_i^l - 1, 0))_{l=1}^{\bar{\tau}-1}) \textit{ otherwise} \end{cases}$$

  where
  - z_i^l denotes the size of the project won by bidder i exactly l periods ago
  - τ_i^l its remaining time until completion
  - For each project l, the size carried over to the next period z_i^{l+1}, equals $(max(\tau_i^l - 1, 0)/\tau_i^l)z_i^l$, and the remaining time until completion τ_i^{l+1} equals $max(\tau_i^l - 1, 0)$.
  - If the contract at period t is won by bidder i, it is added at the beginning of the list of projects with the initial size and time until completion taken from the contract's characteristics $(z_0, \tau_0) \in s_0$.
- $\beta \in (0, 1)$ firm's patience with regard to future profits

The Bidding Model:

- We consider sequential equilibria and restrict our attention to symmetric stationary Markovian strategies. Stationary Markovian strategies do not depend on time. The future looks the same whether the agent is in state s_t at time t or in state s_{t+k} at time $t+k$ provided that $s_t = s_{t+k}$

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- $b(c_i, s_0, s_i, s_{-i})$: A strategy for bidder i is a function of bidder i's cost, the contract's characteristics, and her own and her competitors' states.

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- $b(c_i, s_0, s_i, s_{-i})$: A strategy for bidder i is a function of bidder i's cost, the contract's characteristics, and her own and her competitors' states.
- Payoff of a regular bidder. The discounted sum of future expected payoffs for bidder i can be written in value function form as

$$\begin{aligned} W_i(s, s_0, c_i, b_{-i}) &= max_b[[b - c_i]Pr(i \;\; wins|b, s_0, s_i, s_{-i}, b_{-i}) \\ &+ \beta \sum_{j=0}^{n} Pr(j \;\; wins|b, s_0, s_i, s_{-i}, b_{-i}) \\ &\times E_0 \int W_i(\omega(s_0, s, j), s_0', c_i', b_{-i}) \\ &\times f(c_i'|s_0', \omega_i(s_0, s, j), \omega_{-i}(s_0, s, j))dc_i'] \end{aligned}$$

where $Pr(i \;\; wins|b, s_0, s_i, s_{-i}, b_{-i})$ denotes the probability that bidder i with state s_i wins contract s_0 given the strategy b_{-i} and the state s_{-i} by other bidders; E_0 denotes the expectation operator with respect to the contract characteristics s_0'; and $j = 0$ indicates that nobody won the auction.

The Bidding Model:

- Any bid exceeding reserve price is rejected.
- Assumption of symmetric bidding strategies conditional on state variables implies that the value function can be written as

$$W(s_i, s_{-i}, s_0, c_i, b_{-i}) = W_i(s, s_0, c_i, b_{-i}) \ \forall \ i$$

- Ex ante value function, value function evaluated before contract characteristics and bidder i's cost are known.

$$V_i(s, b_{-i}) = E_0 \int W_i(s, s_0, c, b_{-i}) f(c|s_0, s_i, s_{-i}) dc$$

  where E_0 denotes the expectation with respect to contract characteristics s_0. V_i reflects the expected profits at the beginning of a period before shocks are realized.
- The above value function leads to the following recursive equation for V_i (eq 2.1)

$$\begin{aligned} V_i(s) &= E_0[\int max_b\{[b - c] Pr(i \ \ wins|b, s_0, s_i, s_{-i}) \\ &+ \beta \sum_{j=0}^{n} Pr(j \ \ wins|b, s_0, s_i, s_{-i}) V_i(\omega(s_0, s, j))\} \\ &\times f(c|s_0, s_i, s_{-i}) dc] \end{aligned}$$

Estimation Method:

- Bid distribution function: distribution function of equilibrium bids of i with state (s_0, s_i, s_{-i}), and the associated derivative at points less than or equal to R is $g(\cdot)$. For fringe bidders G_f and g_f
- The probability that i wins with bid b $(Pr(i \;\; wins|s_0, s_i, s_{-i}))$ equals $\Pi_{j \neq i}[1 - G(b|s_0, s_j, s_{-j})]$ where j includes the seller and other regular and fringes bidders.
- We observe data on bids, contract characteristics, and bidders' state variables
- Goal: Infer privately known costs
- The method requires the assumption that observed bids are generated by equilibrium play and satisfy the FOC of optimal bids
- FOC: Let $\phi(\cdot)$ denote the unobserved cost associated with a bid, which is a function of the bid b, the contract characteristics s_0, the state vector (s_i, s_{-i}), and the value function V_i.
- Let

$$h(.|s_0, s_i, s_{-i}) = \frac{g(.|s_0, s_i, s_{-i})}{1 - G(.|s_0, s_i, s_{-i})}$$

denote the hazard function of bids submitted by bidder i when the state equals (s_0, s_i, s_{-i}) Def

Estimation Method:

- The FOC for optimal bids yields the following equation for privately known costs ϕ (eq 3.1)

$$\phi(b|s_0, s_i, s_{-i}, h, V_i) = b - \frac{1}{\sum_{j\neq i} h(b|s_0, s_j, s_{-j})} + \beta \sum_{j\neq i} \frac{h(b|s_0, s_j, s_{-j})}{\sum_{l\neq i} h(b|s_0-, s_l, s_{-l})} [V_i(w(s_0, s, i)) - V_i(w(s_0, s, j))]$$

  that depends on the bid, the hazard function of bids h, and the value function V_i

- The cost equals the bid minus a mark-down.
- The mark-down has two components:
  1. 1rst accounts for the level of competition in the current period.
  2. 2nd, accounts for the the incremental effect on the future discounted profit if firm i wins the contract instead of another firm
- For fringe bidders, they assign no value to the future $V_f = 0$ and the second term vanishes

Estimation Method:

- In order to infer the distribution of costs, we need estimators for:
  - The transition function of the state ($\omega(s_0, s, j)$). It is a given function
  - The bid hazard function ($h(.|s_0, s_i, s_{-i})$). Obtained from the data on bids, contract characteristics and state variables
  - The secret reserve price hazard function. Not estimated due to the lack of data on rejected bids. Assumed to follow a uniform distribution.
  - The discount factor (β). Treated as given. Check how sensitive the estimates are to changes in β
  - The value function ($V_i(s)$). It involves cost variables that are unobserved, and decisions by multiple agents, which are endogenous.

Estimation Method: The Value Function

- The key element of the method is the fact that the distribution of equilibrium bids determines the discounted sum of expected future profits. Thus, there is a representation of the value function in terms of the distribution of bids only.
- **Conditional on the states, the bidder chooses the action b that yields the highest expected net present value of operating profits. The bidder updates its state variables and made another decision at t+1**

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- **Proposition 1: (eq 3.2)**

$$\begin{aligned} V_i(s) &= E_0\{\int_{\underline{b}}^{R} \frac{1}{\sum_{j\neq i} h(\cdot|s_0, s_j, s_{-j})} dG^{(i)}(.|s_0, s) \\ &+ \beta \sum_{j=0, j\neq i}^{n} [Pr(j \quad wins|s_0, s_i, s_{-i}) + \int_{\underline{b}}^{R} \frac{h(.|s_0, s_i, s_{-i})}{\sum_{l\neq i} h(.|s_0, s_l, s_{-l})} dG^{(j)}(.|s_0, s)] \\ &\times \quad V_i(\omega(s_0, s, j))\} \end{aligned}$$

The value function has two parts:

1. 1st accounts for bidder i's current expected profits.
2. 2nd part accounts for bidder i's sum of discounted expected future payoffs.

- Each bidder dynamic game is reduced to a single agent dynamic decision problem where each bidder maximizes the discounted sum of future payoffs taking as given the equilibrium bid distribution associated with other bidders.

Approximation of the value function:

- Discretize observed state variables (instead of forward simulation)
- We select a grid of state vectors $\hat{S} = (s^1, ..., s^m)$ by drawing 50 states from the distribution of observed states. We numerically solve eq 3.2 for every bidder on this grid.

$$A_i(s) = E_0\{\int_{\underline{b}}^{R} \frac{1}{\sum_{j \neq i} h(.|s_0, s_j, s_{-j})} dG^{(i)}(.|s_0, s)\}$$

and the $1 \times m$ vector of transition probabilities of the events that the states $(s^1, ..., s^m)$ are reached when bidder j wins the contract

$$B_{ij}(s) = E_0\{\int_{\underline{b}}^{R} [1 + \frac{h(.|s_0, s_i, s_{-i})}{\sum_{l \neq i} h(.|s_0, s_l, s_{-l})}] dG^{(j)}(.|s_0, s) \times (1_{\hat{\omega}(s_0,s,j)=s^1}, ..., 1_{\hat{\omega}(s_0,s,j)=s^m})\}$$

In both expressions, the first expectation is with respect to contract characteristics.

- We evaluate the expectation wrt contract characteristics in the following way: we select a set of contract chars $\hat{S}_0$ by randomly drawing 50 contracts from the set of observed contracts and use the sample average.

Approximation of the value function:

- We evaluate the expectation wrt the bid distribution function by numerical integration using the estimated derivatives $d\hat{G}^{(i)}$ and $d\hat{G}^{(j)}$.
- Using the symbol A_i for current period payoff and B_{ij} for the transition probabilities, the value function is given by

$$V_i(s) = A_i(s) + \beta \sum_{j \neq i} B_{ij}(s) V_i$$

where V_i denotes the vector $(V_i(s^1), ..., V_i(s^m))'$

- The value function can be expressed as

$$V_i = [I - \beta B_i]^{-1} A_i$$

Estimation of the cost distribution function

- The relationship between the distribution function of costs and the distribution function of bids is given by $F(c|s_0, s_i, s_{-i}) = G(b(c, s_0, s_i, s_{-i})|s_0, s_i, s_{-i})$.

- The inverse of the bid function conditional on state variables, $c = \phi(b|s_0, s_i, s_{-i})$ is given in eq 3.1.

Bid distribution functions

- Estimates of the distribution of equilibrium bids ($g()$, $G()$)

Bid distribution functions

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- Parametric specification vs NonParametric

- Density function of regular bidders: Use Weibull density. ▸ Link

$$g(b|\theta_1, \theta_2, \theta_3) = \frac{1}{b+1}[\frac{\theta_1[ln(b+1) - ln(\theta_3+1)]^{\theta_1 - 1}}{\theta_2^{\theta_1}}]exp(-(\frac{ln(b+1) - ln(\theta_3+1)}{\theta_2})^{\theta_1})$$

  The support of bids for regular bidders is $[\theta_3, \infty)$. The parameters depend on state variables.

- The density function of bids by fringe bidders follows a beta distribution. ▸ Link

$$g_f(b|\theta_3, \theta_4, \theta_5) = \frac{1}{R-\theta_3}(\frac{b-\theta_3}{R-\theta_3})^{\theta_4 - 1}(\frac{R-b}{R-\theta_3})^{\theta_5 - 1}\frac{1}{B(\theta_4, \theta_5)}$$

  the support of fringe bids is $[\theta_3, R]$, where R is the upper bound on the floor

- θ depends on states

Bid distribution functions: Likelihood function

- Bid data for regular bidders are censored. We only observe bids that are below the upper bound of the reserve price R^t
- The likelihood of regular and fringe bids conditional on the initial state s^0

$$L = \Pi_t[\Pi_{i=1}^{n_r}[g(b_i^t|\theta^{it})]^{o_i^t}[1 - G(R^t|\theta^{it})]^{1-o_i^t}\Pi_{j=n_r+1}^{n^t} g_f(b_j^t|\theta^t)]$$

  where

  - b_i^t is the bid by regular bidder i on contract t
  - b_j^t is the bid by fringe bidder j on contract t
  - o_i^t be a dummy variable that equals one if we observe a bid by bidder i on contract t, and zero otherwise.
  - θ depends on states

- We maximize the logarithm of the likelihood function.

$$\theta^* = argmax_\theta logL$$

Estimation Results: Bid Distribution Functions

- The estimates suggests that the presence of capacity constraints play an important role in highway bidding

TABLE IV
PARAMETER ESTIMATES OF THE BID DISTRIBUTIONS

Data:	Regular and Fringe Bids			Regular and Fringe Bids		
Number of Observations:	30,873			30,873		
Log Likelihood:	−1,467.46			−1,500.82		
Variables	Regular Bids	Fringe Bids	Lower Bound	Regular Bids	Fringe Bids	Lower Bound
θ_1	−13.1688			−13.1721		
	(22.689)			(14.779)		
θ_4		−1.3354			−1.3357	
		(0.155)			(0.154)	
Constant	5.6934	1.8821	−0.8228	5.6997	1.8820	−0.8228
	(0.193)	(0.170)	(0.019)	(0.193)	(0.151)	(0.022)
Ln Estimate	−0.3739	−0.0014	1.0541	−0.3767	−0.0014	1.0541
	(0.015)	(0.011)	(0.001)	(0.015)	(0.006)	(0.001)
Ln Working Days	0.2765	−0.0283	−0.0049	0.2813	−0.0283	−0.0049
	(0.021)	(0.018)	(0.002)	(0.021)	(0.019)	(0.002)
Estimate/Reserve_Price	−2.8037	−2.5396	−0.0188	−2.7803	−2.5395	−0.0188
	(0.200)	(0.158)	(0.013)	(0.200)	(0.158)	(0.013)
Nbid-Fringe	0.025	0.0049	0.0011	0.0258	0.0049	0.0011
	(0.006)	(0.008)	(0.001)	(0.007)	(0.006)	(0.001)
Distance	0.0559			0.0562		
	(0.003)			(0.003)		
# of Plants within the Region	−0.3144			−0.3159		
	(0.026)			(0.027)		
Backlog	0.0721			0.0803		
	(0.015)			(0.016)		
Sum_Distance	0.0015	0.0002	−0.0004	0.0015	0.0002	−0.0004
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)
Sum_# of Plants within the Region	0.1242	−0.0262	−0.0129	0.1271	−0.0262	−0.0129
	(0.012)	(0.013)	(0.002)	(0.012)	(0.014)	(0.002)
Sum_Backlog	0.0108	0.0278	−0.0022	0.0128	0.0277	−0.0022
	(0.006)	(0.007)	(0.001)	(0.006)	(0.007)	(0.001)

Estimation Results: Bid Distribution Function

- Figure 1 shows the bid distribution function between the lower bound of bids and the floor, evaluated at sample average values of state variables.
- On avg, unconstrained bidders are about twice as likely to submit a bid than constrained bidders. This confirms the importance of capacity constraints
- The probability of submitting a bid decreases monotonically in backlog, which is consitent with the notion of capacity constraints.

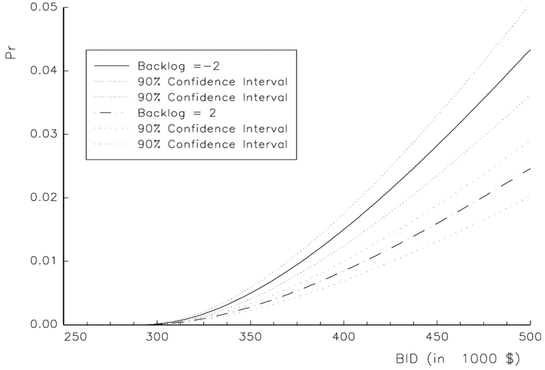

FIGURE 1.—Bid distribution function.

Estimation Results: Estimates of the Value Function

- The state variables entering the value function are: bidder i's backlog and three vars for each competitor consisting of the competitor's backlog, the competitor's backlog interacted with average distance, and the competitor's backlog interacted with average number of plants per region
- The plot illustrates the discounted expected future profit of bidder 3 by varying the backlog var of bidder 3 between -1.6 and 1.6. the competitors' state variables are fixed at their sample averages. We impose β equal 0.8.

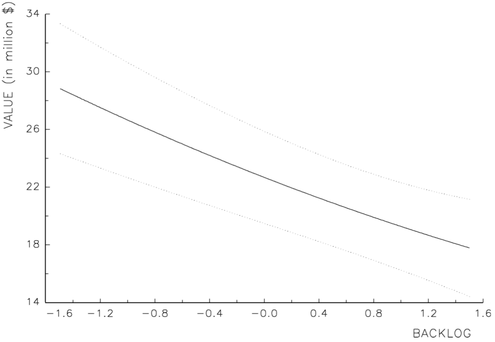

FIGURE 2.—Value function.

- The avg discounted sum of payoffs for bidder 3 equals \$25 million. Other regular bidders' discounted sum of payoffs ranges between \$7 and \$73 million.
- The effect of backlog on the value function in fig 2 is negative, which is in accordance with the expected effect under capacity constraints. In Figure 2, backlog reduces the value function in total by about 35%.

Estimates of costs: Equilibrium bid function

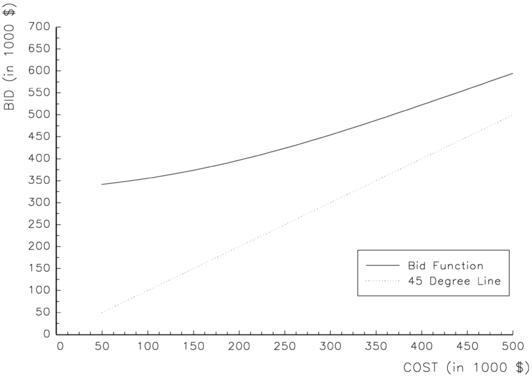

FIGURE 3.—Bid function.

- Bid function for bidder 3. The bid function is estimated using eq 3.1 and plotted by fixing the state vars at sample average values for bidder 3 and varying the cost.
- The markup denotes the difference between the bid and the cost of a bidder.
- A substantial portion of the markup of regular bidders is attributable to the loss in future discounted value due to limited capacity. This loss reflects the cost of winning today vs winning later. We can measure this loss based on eq 3.1, which decomposes the mark-up into two parts: the first part reflects contemporaneous competition. The second part measures the loss in value of winning today vs winning later. **For bidder 3, on avg, across all obs bids, 64.2% of the markup is attributable to the second part, which is the option value of winning today vs winning later.**

Estimation Results: Estimates of costs

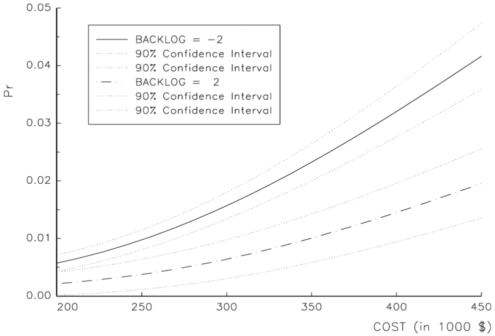

FIGURE 4.—Cost distribution function.

- Figure depicts the distribution function of costs for bidder 3. Distributions are reported for two values of backlog and holding the other state variables at sample average values.
- On average the probability that the cost is below a certain threshold is more than twice when the bidder is unconstrained than when the bidder is constrained.

The effect of backlog and inefficiencies: Inefficiencies

- The benefits of an additional unit of capacity is an important part in the eq that determines the optimal capacity choice as a function of the returns to capacity and the cost of investing in capacity. (allocation of budgets across different campaigns)
- Due to the presence of inter-temporal effects and due to bidder heterogeneity, a first-price auction need not select the efficient firm. The bidder with the lowest bid need not be the bidder with the lowest cost. The reason is that constrained (or smaller) bidders may bid more aggressively than unconstrained (or larger) bidders. The strategic bid shading can imply that a constrained firm wins although it did not have the lowest cost

TABLE V
ESTIMATES OF EFFICIENCY LOSSES

	Range of Engineers' Estimate (in $ 1,000)					
Variable	[0, 100]	(100, 400]	(400, 1000]	(1000, 5000]	(5000, .]	Overall
All Contracts:						
Number of Contracts	49	893	525	545	184	2196
Prob of an Inefficiency	0.02	0.52	0.17	0.24	0.11	0.32
Average Efficiency Loss[a]	0.04	0.08	0.14	0.17	0.19	0.13
Contract Won by a Regular Bidder:						
Number of Contracts	1	90	94	147	42	374
Prob of an Inefficiency	0.00	0.20	0.55	0.38	0.27	0.36
Average Efficiency Loss[a]	0.36	0.29	0.29	0.22	0.24	0.25
Contract Won by a Fringe Bidder:						
Number of Contracts	48	803	431	398	142	1822
Prob of an Inefficiency	0.02	0.56	0.08	0.19	0.06	0.31
Average Efficiency Loss[a]	0.04	0.06	0.11	0.15	0.18	0.10

[a]Efficiency losses are reported as a fraction of the engineers' estimate.

Conclusions

- This paper proposes an estimation method for a repeated auction game under the presence of capacity constraints and bidder asymmetry
- use highway procurement auctions in CA
- We characterize costs as a function of state variables and illustrate the bidding equilibrium
- Bidders that have a large fraction of their capacity committed have on average higher costs than bidders with little capacity committed
- We find that when all bidders are capacity constrained, the resulting price paid by the auctioneer is about 18% higher than when all regular bidders are unconstrained.
- Two policy implications:
  1. Scheduling and timing of contracts offered for sale influences the final price. Thus, optimal scheduling taking the endogeneity of capacity choices into account may save costs.
  2. Due to inter-temporal constraints and bidder heterogeneity, an inefficient firm may be chosen. Our experiment indicates that inefficiencies may arise on about 32% of all contracts and they may amount to 13% of the expected contract size. Our estimates suggest that auction rules that cope better with inter-temporal effects and bidder asymmetry could be a cost saving alternative
- There are two shortcomings of our estimation method: first, our estimation method relies on the assumptions that bidders completely understand the auction environment and that our estimates of winning probas correctly capture bidders' perceived winning odds.

Definitions

- Hazard Function: The probability of observing an outcome within the neighborhood of of x, conditional on the outcome being no less than x back