Empirical Auctions Models

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Introduction: References

- John Asker, Matthew Shum, Jon Levin Lecture Notes
- Hickman, Hubbard, Saglam: "Structural Econometric Methods in Auctions: A Guide to the Literature"
- Athey and Haile: "Empirical Models of Auctions"
- Athey and Haile: "Nonparametric Approach to Auctions"

Introduction: References

Reasons for empirical work on auctions

- Validating basic assumptions (the role of asymmetric information - Hendricks and Porter (1988))
- Testing theory: theory makes predictions about how model primitives map to outcomes (Handbook of Experimental Economics)
- Evaluating policy: the optimality of design decisions depends on the properties on the underlying primitives.
    - Uncovering the specific distribution of private information
    - Uncovering the properties of the structure of private information
- Objective: Uncover the underlying distribution of private information.

Introduction: References

- Early descriptive work in 1960s and 1970s describing features of bidding for treasure bills, oil leases, timber in national forests
  - Johnson (1979), Hansen (1986) use change in US Forest Service policy to compare revenue in open and sealed bid auctions - results are inconclusive
- Hendricks and Porter (1988) use Milgrom-Weber theory of common value auctions with an informed and uninformed bidder to analyze behavior in "drainage tract" oil lease auctions.
  - They show that bidders owning neighboring tracts make much higher expected profit than de novo bidders with potentially less information.
  - Athey and Levin (2001) use ex post data to test for presence of asymmetric information in timber auctions, and identify gaming of auction rules.

Introduction: References

- Paarch's (1992) Stanford dissertation estimates parametric IPV and CV sealed tender models and tests between them.
- Laffont, Ossard and Vuong (1995) show how prices from an ascending auction data can be used to estimate bidder value distributions.
- Guerre, Perrigne and Vuong (2000) show how bid data from sealed bid auctions can be "inverted" to recover bidder value distributions.
- Dozens of papers follow develop and extend this idea to ascending auction data, multi-unit auctions, studies of collusion, market power, etc...

Reduced Form: Henricks and Porter (FPSB)

- Henricks and Porter (1988): "An Empirical Study of an Auction with Assymetric Information" - Mineral Rights Model
- Setting: Drainage leases in OCS 1959 - 69 - leases next to tract in which an oil deposit has been discovered
- Symmetry / Asymmetry of information is important for qualitative predictions in CV auctions. Drainage vs Wildcat: drainage is adjacent to known deposit, wildcat is not.
- Research Question: Does the bidding behavior look consistent with a CV model that reflects institutions? Is there evidence of bidding coordination?
- Conclusion: Data are consistent with the predictions of the Bayesian Nash Equilibrium model of bidding in first-price, sealed bid auction with asymmetric information

Reduced Form: Henricks and Porter (FPSB)

TABLE 1—SELECTED STATISTICS ON WILDCAT AND DRAINAGE TRACTS[a]

	Wildcat	Drainage
Number of Tracts	1056	144
Number of Tracts Drilled	748	124
Number of Productive Tracts	385	86
Average Winning Bid	2.67	5.76
	(0.18)	(1.07)
Average Net Profits	1.22	4.63
	(0.50)	(1.59)
Average Tract Value	5.27	13.51
	(0.64)	(2.84)
Average Number of Bidders	3.46	2.73

[a]*Source:* Kenneth Hendricks, Robert Porter, and Bryan Boudreau (1987). Dollar figures are in millions of $1972. The numbers in parentheses are standard deviations of the sample means.

Reduced Form: Henricks and Porter (FPSB)

- Facts:
  - More than twice the average value of wildcat tracts
  - There was less competition, and profit was roughly four times higher on drainage tract than on wildcat tract
  - Even though drainage tracts had lower risk investments and yielded a significantly higher rate of return, firms were less likely to participate in these auctions.
- The main difference between wildcat and drainage auctions is the distribution of information.
- Neighbor firms likely to be better informed than non-neighbor firms, which, if true, would give them an advantage in bidding agains the latter (Winner's Curse).

Reduced Form: Henricks and Porter (FPSB)

- Neighbor firms likely to be better informed than non-neighbor firms, which, if true, would give them an advantage in bidding agains the latter (Winner's Curse).
- Model:
  - Participation and bidding decisions of neighbor firms are better predictors of tract profitability than the ones of non-neighbor firms.
  - Neighbor firms won most of the profitable drainage tracts. By contrast, non-neighbor firms earned approx zero profits.
- Data are consistent with predictions of the Bayesian Nash equilibrium model of bidding in first-price, sealed bid auction with asymmetric information.

Reduced Form: Henricks and Porter (FPSB)

- Solve by looking for BNE
- Comparative statics that come out of the model that are taken to data:
  - The event that no neighbor firms bids occurs less frequently than the event that no non-neighbor firm bids
  - The neighbor firm wins at least one half of the tracts
  - Expected profits to non-neighbor firms are zero. They are negative on the set of tracts where no neighbor firm bids, and positive on the set of tracts where the neighbor firm bids
  - Expected profits to the neighbor firm incorporates an information premium which makes its earnings above "average"
  - The bidding strategy of the neighbor firm is independent of the number of non-neighbor firms
  - The bidding strategy of the neighbor firm is an increasing function of the public signal, when a larger signal is "good news"

Reduced Form: Henricks and Porter (FPSB)

Data

- Federal lands off the coasts of Louisiana and Texas which were leased between 1959 and 1969.
- The government auctioned 144 tracts
- Each lease is sold via first-price, sealed bid auction.
- The government may participate in the auction in two ways:
  - Reservation price (around $25 per acre)
  - Right to reject the high bid on a tract if it believes the bid is too low

Reduced Form: Henricks and Porter (FPSB)

- Distribution of neighbor firms per drainage tract and dist. of bids

	0	1	2	3	4	5	6
No. of tracts	0	40	43	21	5	4	1

FIGURE 1. NUMBER OF NEIGHBOR FIRMS

	0	1	2	3	4	5	6
No. of tracts	19	79	15	0	0	1	0

FIGURE 2. NUMBER OF NEIGHBOR BIDS

TABLE 2—DEFINITION OF VARIABLES[a]

	Mean	Standard Deviation
B_I: maximum bid by neighbor	3.78	11.52
B_U: maximum bid by non-neighbor	3.60	9.57
N_I: number of neighbor bids	1.00	0.67
N_U: number of non-neighbor bids	1.69	2.09
N: number of neighbor tracts	3.01	1.98
NF: number of neighbor firms	2.06	1.08
π: *ex post* tract gross profitability	8.75	20.83
V: *ex post* gross profits of adjacent tract	14.51	20.16
A: tract acreage	2.679	1.533

[a] Dollar figures are in millions of $1972. Tract acreage is in thousands of acres.

- PI of the neighbor firms is the gross profits of the tract (π)

Reduced Form: Henricks and Porter (FPSB)

TABLE 3—SAMPLE STATISTICS ON TRACTS WON BY EACH TYPE OF FIRM[a]

	Wins by Neighbor Firms		Wins by Non-Neighbor Firms		
	A	Total	B	C	Total
No. of Tracts	35	59	19	36	55
No. of Tracts Drilled	23	47	18	33	51
No. of Productive Tracts	16	36	12	19	31
Average Winning Bid	3.28	6.04	2.15	6.30	4.87
	(0.56)	(2.00)	(0.67)	(1.31)	(0.92)
Average Gross Profits	10.05	12.75	−0.54	7.08	4.45
	(3.91)	(3.21)	(0.47)	(2.95)	(1.99)
Average Net Profits	6.76	6.71	−2.69	0.78	−0.42
	(3.02)	(2.69)	(0.86)	(2.64)	(1.76)

[a] Dollar figures are in millions of $1972. The numbers in parentheses are the standard deviations of the sample means. Column A refers to tracts which received no non-neighbor firm bid, column B refers to tracts which received no neighbor bid, and column C to those in which a neighbor firm bid, but a non-neighbor firm won the tract.

Reduced Form: Henricks and Porter (FPSB)

- Bidding Coordination

TABLE 4—THE EFFECT OF NEIGHBOR FIRM COMPETITION ON NEIGHBOR FIRM PARTICIPATION AND PROFITS[a]

	Single Neighbor Tracts	Multiple Neighbor Tracts No. of Neighbor Bids		
		1	≥ 2	Total
No. of Tracts	40	48	15	74
No. of Tracts with No Neighbor Bid	8	–	–	11
No. of Wins	19	29	11	40
Average Winning Bid	4.795	2.615	17.193	6.624
of Neighbor Firm	(1.444)	(0.697)	(9.953)	(2.885)
Average Gross Profits	13.601	4.670	32.597	12.350
of Neighbor Firm	(5.608)	(2.148)	(11.506)	(3.965)
Average Net Profits	8.806	2.055	15.404	5.725
of Neighbor Firm	(4.762)	(1.690)	(10.963)	(3.297)

[a] Dollar figures are in millions of $1972. The numbers in parentheses are the standard deviations of the sample means.

Reduced Form: Henricks and Porter (FPSB)

Likelihood Function

- Reduce Form Equation

$$Y_{it} = W_{it}\theta_i + \epsilon_{it} \;\; i = I, U \;\; t = 1, ..., T$$

where W_{it} is a vector of regressors for tract t, $\{\epsilon_{It}, \epsilon_{Ut}\} \sim \mathcal{N}(0, \{\sigma_I^2, \sigma_u^2, \sigma_{IU}\})$

- Dependent variable Y_{it}

$$log(B_{it}|R_t) = Y_{it} \;\; if \;\; Y_{it} \geq 0, \;\; 0 \;\; Otherwise$$

where R_t is the reservation price on tract t

- Bids are assumed to be log normally distributed

Reduced Form: Henricks and Porter (FPSB)

Likelihood Function

- Log Likelihood Function

$$Log \;\; L = \sum_{t \in \Omega_{++}} l_{1t} + \sum_{t \in \Omega_{+0}} l_{2t} + \sum_{t \in \Omega_{0+}} l_{3t}$$

where

$$\begin{aligned} l_{1t} \;\; = \;\; & -[log(2\pi) + (1/2) log|\Sigma|] - (1/2)(\epsilon_{It}, \epsilon_{Ut})\Sigma^{-1}(\epsilon_{It}, \epsilon_{Ut})' \\ & -log(1 - Z(-W_{Ut}\theta_U/\sigma_U, W_{It}\theta_I/\sigma_I; \rho_{IU})) \end{aligned}$$

- Estimation

$$\min_{\theta,\sigma} -Log \;\; L$$

Reduced Form: Henricks and Porter (FPSB)

- Theory: Conditioning solely on public info. the dist. of the informed bid and maximum uninformed bid should be approx. the same

TABLE 6—JOINT DISTRIBUTION OF BIDS CONDITIONAL ON PUBLIC INFORMATION[a]

Independent Variable	Unrestricted: Dependent Variable $\log(B_I/R)$	Unrestricted: Dependent Variable $\log(B_U/R)$	Restricted: Dependent Variable $\log(\text{BID}/R)$
Constant	1.98068	2.05437	−1.99365
	(3.44)	(2.70)	(3.96)
V	0.07391	0.00523	0.04966
	(3.42)	(0.19)	(2.52)
V^2	−0.00073	−0.00009	−0.00050
	(−2.92)	(−0.30)	(−2.17)
A	−0.11092	0.13285	−0.02499
	(−0.82)	(0.74)	(−0.21)
N	−0.08226	−0.28903	−0.14763
	(−0.74)	(−1.97)	(−1.51)
$\begin{bmatrix} \sigma_I & \\ \rho_{IU} & \sigma_U \end{bmatrix}$	$\begin{bmatrix} 2.0151 & \\ (11.7) & \\ 0.1034 & 2.6596 \\ (0.94) & (12.7) \end{bmatrix}$		$\begin{bmatrix} 2.0528 & \\ (11.5) & \\ 0.0638 & 2.6785 \\ (0.57) & (12.8) \end{bmatrix}$
	Log $L = -428.895$		Log $L = -434.184$

[a]Asymptotic t-statistics are in parentheses. They are computed from the analytic second derivatives. They are not appreciably different from the Eicker-White t-statistics.

Reduced Form: Henricks and Porter (FPSB)

- Theory: Conditioning solely on public info. and PI (tract profits) the dist. of the informed bid and maximum uninformed bid should differ

TABLE 7—JOINT DISTRIBUTION OF BIDS CONDITIONAL ON PUBLIC INFORMATION AND TRACT PROFITABILITY[a]

Independent Variable	Unrestricted: Dependent Variable $\log(B_I/R)$	Unrestricted: Dependent Variable $\log(B_U/R)$	Restricted: Dependent Variable $\log(\text{BID}/R)$
Constant	1.86237	2.07435	1.88962
	(4.15)	(2.42)	(4.63)
π	0.09102	0.02765	0.07532
	(4.30)	(0.79)	(3.93)
π^2	−0.00053	−0.00026	−0.00046
	(−2.12)	(−0.62)	(−2.09)
V	0.04428	−0.00323	0.03361
	(2.55)	(−0.12)	(2.05)
V^2	−0.00045	−0.00001	−0.00036
	(−2.25)	(−0.03)	(−1.80)
A	−0.20962	0.10221	−0.13419
	(−1.95)	(0.58)	(−1.34)
N	−0.00888	−0.25858	−0.06645
	(−0.10)	(−1.81)	(−0.83)

$$\begin{bmatrix} \sigma_I & \\ \rho_{IU} & \sigma_U \end{bmatrix}$$

Unrestricted: $$\begin{bmatrix} \underset{(11.5)}{1.5996} & \\ \underset{(0.46)}{0.0492} & \underset{(13.1)}{2.6162} \end{bmatrix}$$ Log $L = -409.0028$

Restricted: $$\begin{bmatrix} \underset{(11.3)}{1.6379} & \\ \underset{(-0.20)}{-0.0216} & \underset{(11.8)}{2.8014} \end{bmatrix}$$ Log $L = -418.9243$

[a]Asymptotic t-statistics are in parentheses. They are computed from the analytic second derivatives.

Reduced Form: Henricks and Porter (FPSB)

- Theory: N_U should have no explanatory power on the informed bid equation

TABLE 8—BID EQUATIONS[a]

Independent Variable	Equation (1) Dependent Variable $\log(B_I/R)$	Equation (1) Dependent Variable $\log(B_U/R)$	Equation (2) Dependent Variable $\log(B_I/R)$	Equation (2) Dependent Variable $\log(B_U/R)$	Equation (3) Dependent Variable $\log(B_I/R)$	Equation (3) Dependent Variable $\log(B_U/R)$
Constant	1.86973	2.13073	1.64933	2.15018	1.67785	0.064395
	(−4.19)	(2.90)	(3.52)	(2.96)	(3.66)	(1.14)
π	0.08967		0.08505		0.08501	
	(4.26)		(4.09)		(4.08)	
π^2	−0.00051		−0.00047		−0.00047	
	(−2.04)		(−1.88)		(−1.88)	
V	0.04452	0.00257	0.04814	0.00120	0.04757	0.02083
	(2.58)	(0.10)	(2.82)	(0.04)	(2.79)	(1.08)
V^2	−0.00045	−0.00006	−0.00047	−0.00005	−0.00046	−0.00011
	(−2.25)	(−0.21)	(−2.47)	(−0.18)	(−2.42)	(−0.58)
A	−0.20738	0.12154	−0.25435	0.12908	−0.25713	−0.22645
	(−1.95)	(0.68)	(−2.32)	(0.74)	(−2.38)	(−1.71)
N	−0.01001	−0.27341	0.03228	−0.27116	0.03506	0.03029
	(−0.12)	(−1.92)	(0.36)	(−1.93)	(0.41)	(0.28)
N_U			0.13505		0.11312	0.83705
			(1.26)		(1.42)	(8.48)
$\begin{bmatrix} \sigma_I & \\ \rho_{UI} & \sigma_U \end{bmatrix}$	1.5956		1.5664		1.5663	
	(11.5)		(11.3)		(11.5)	
	0.0453	2.6238	−0.0782	2.6101	−0.0576	1.8769
	(0.43)	(13.0)	(−0.62)	(13.0)	(−0.56)	(13.0)
	Log $L = -409.3745$		Log $L = -408.6295$		Log $L = -378.5628$	

[a]Asymptotic t-statistics are displayed in brackets. They are computed from the analytic second derivatives.

Structural Models: Guerre, Perrigne, and Vuong (1995)

FPSB with Independent Private Values

- Bidders $1, ..., N$ draw independent private values from F
- Data consist of bids $b_{1t}, \cdots, b_{Nt}$ from T auctions
- Define:

$$G_i(b) = Pr(max_{j \neq i} b_j \leq b_i) = Pr(b_i \text{ is winning bid}) = F(v)$$

- Bidder i's problem:

$$max_{b_i}(v_i - b_i)G_i(b)$$

- In equilibrium, we must have:

$$b_i = v_i - \frac{G_i(b_i)}{(n-1)g_i(b_i)}$$

Structural Models: Guerre, Perrigne, and Vuong (1995)

FPSB with Independent Private Values

- Data consist of bids $b_{1t}, \cdots, b_{Nt}$ from T auctions
- Fix a bidder i. Use observed bids to construct

$$G_i(b) = Pr(max_{j \neq i} b_j \leq b_i) = \Pi_{j \neq i} Pr(b_j \leq b_i | X_t)$$

- Use equilibrium condition to recover v_i's

$$\hat{v}_{it} = b_{it} + \frac{\hat{G}_i(b_i)}{(n-1)\hat{g}_i(b_i)}$$

- The RHS can be estimate from the data: G and g can be estimated nonparametrically
- Does not require bidder symmetry, and can be extended to allow each auction to have different "characteristics" x_t, so $\hat{G}_i(b_i|x_t)$, or to allow for correlated bids/values.

Structural Models: Guerre, Perrigne, and Vuong (1995)

FPSB with Independent Private Values

$$\hat{g}(b) = \frac{1}{T \times n} \sum_{t=1}^{T} \sum_{i=1}^{n} \frac{1}{h} \mathcal{K}(\frac{b - b_{it}}{h})$$

$$\hat{G}(b) = \frac{1}{T \times n} \sum_{t=1}^{T} \sum_{i=1}^{n} 1(b_{it} \leq b)$$

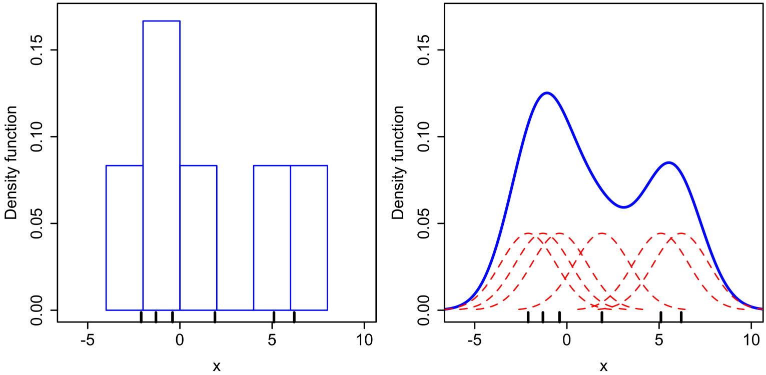
where $\mathcal{K}$ is a kernel function (for instance normal). Hence, Guerre, Perrigne, and Vuong recommend a two-step approach to estimate the valuation distribution $f(v)$

1. In the first step, estimate $G(b)$ and $g(b)$ nonparametrically
2. In the second step, estimate $f(u)$ by using kernel density estimator of recovered valuations

$$\hat{f}(v) = \frac{1}{T \times n} \sum_{t=1}^{T} \sum_{i=1}^{n} \frac{1}{h} \mathcal{K}(\frac{v - \hat{v}_{it}}{h})$$

Structural Models: Guerre, Perrigne, and Vuong (1995)

Normal Kernel



Structural Models: Guerre, Perrigne, and Vuong (1995)

Bandwith