IMMUNIZED INTERNATIONAL ALLIANCES: 
A SEQUENTIAL GAME OF ALLIANCE FORMATION 
IN THE AIRLINE INDUSTRY

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Abstract. This paper analyzes the incentives of competing airlines to form international alliances, and the concerns that competition authorities may have in granting such agreements. We consider a sequential game where a carrier (leader) first decides if it wants to establish a complementary international alliance with a foreign carrier. This agreement improves partners’ coordination on international routes where both operate, letting them benefit from joint fares and capacity on these routes. Then, a competitor (follower) determines whether allying with another foreign carrier is its best response to the action taken by the leader. The alliance affects not only the international markets where airlines coordinate, but also the nature of competition in domestic markets. Several equilibria arise depending on the strength of the brand loyalty of travelers, economies of traffic density, and synergies derived from the alliance. Welfare analysis suggests that when forming an international alliance belongs to the set of equilibria, total surplus does not always increase. In some cases the leader decides to coordinate even if the alliance creates negative synergies, with the objective of deterring the alliance formation of the follower. In this scenario, competition authorities should be concerned about granting antitrust immunity to the alliance.

Keywords: Airline Industry, Antitrust Immunity, Antitrust Policy, Industrial Organization

1. Introduction:

International alliances in the airline industry are a common practice nowadays. By far, the most controversial type is the one created when competition authorities grant antitrust immunity to allied carriers (ATI). Granting exemption from antitrust laws allows participants

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to agree on prices and capacity. The scope of the agreement is determined by the interest of airlines and limited by competition authorities. ATI may have several pro-competitive and anti-competitive effects and depending on which effects dominate, granting an ATI can be desirable from a welfare point of view.

We present a theoretical model of alliance formation where we examine the strategic behavior of airlines as a reaction to decisions taken by competitors. The main paper’s contribution is to provide an analysis of the incentives of competing airlines to create rival alliances using an extension of the model presented by Brueckner and Whalen (2000), and the welfare impact of international agreements. We consider a sequential game where there is a leader (airline 1) which first decides if it wants to form an alliance with a carrier operating in another country and ask for ATI. If ATI is granted by competition authorities, allied carriers are allowed to jointly determine the price of their flights operating in international markets and share the revenues derived from them. Then, in a second stage of the game, the competitor (airline 2) decides whether an agreement with another foreign carrier is its best response to the action taken by the leader. Finally, airlines simultaneously decide on prices. We can imagine the leader as a carrier which has already a tight relationship with airlines operating in the other country (for instance, codesharing or joint frequent flyer programs). In such a case, it would be easier for it to establish an alliance with a foreign airline than for airlines without such ties (follower). We will see that these decisions not only have consequences on the international routes, but also on domestic markets.

The model captures three important features of the airline industry: travelers brand-loyalty, existence of economies of traffic density, and synergies from airline cooperation. The first effect accounts for preferences of travelers for one carrier, the second takes into account decreasing marginal costs, and the third is related to possible efficiency gains (or losses) derived from sharing activities. Airline cooperation may reduce fixed costs of partners (e.g. more efficient scheduling or integration of ground activities), or increase them (e.g. cost of coordination). Moreover, we assume that such synergies may be asymmetric across airlines. While previous theoretical work has considered the existence of brand-loyalty and economies of traffic density, we are not aware of any study that captures synergies as a result of alliance formation.

The main results of the paper are: first, ATI not only affects the international markets where airlines coordinate, but also the strategic behavior of carriers in domestic markets. Second, if low brand loyalty of travelers is accompanied by high economies of traffic density and positive synergies, airlines may find it profitable to ally. However, if carriers benefit from very loyal customers and the magnitude of economies of traffic density and synergies are not
high enough, carriers may find it desirable to operate on their own rather than sharing part of their profits with a partner. Third, we can have asymmetric decision outcomes where a company may be willing to ally and the competitor does not find it profitable. Finally, welfare analysis suggests that if asking for ATI belongs to the set of equilibria, total surplus does not always increase. In some cases, the leader decides to coordinate even if the alliance creates negative synergies, with the objective of deterring the alliance formation by the follower. In this scenario, competition authorities should be concerned about granting ATI to the leader.

ATI may have several pro-competitive and anti-competitive effects. However, empirical evidence from the US-Europe Transatlantic market suggests, overall, a positive impact of ATI on consumer welfare. Good examples are the reports presented by the US Department of Transportation (1999, 2000). Such studies concluded that antitrust immunity expands the service offered by carriers and does not significantly increase fares in markets where immune alliances operate. Similar conclusions have been derived by other authors such as Brueckner (2003), Whalen (2007) and Willig, Israel and Keating (2009).

There exist several reasons that justify granting ATI. First, the airline industry benefits from economies of traffic density (Brueckner and Spiller 1994). This effect reduces the carrier marginal cost when the number of passengers increases. The greater the network, the greater is the impact of this effect. Second, when passengers need to use several carriers to reach their international destination, the joint fare offered by the alliance eliminates double marginalization (Brueckner 2001). That is, cooperative pricing internalizes the negative externality that arises when each partner chooses its own subfare without taking into account the consequences for the other carrier. Third, alliances offer “seamless services” by increasing scheduling coordination for connecting flights and integration of ground activities. Additionally, customers can benefit from attractive frequent flyer programs, due to the incremental destinations offered by the agreement. Finally, international alliances can also be seen as an alternative to overcome restrictions in some markets. For example, some countries have bilateral agreements that limit the number of carriers and airports operating between both countries. In some other cases, regulations limit the ownership by foreign investors of domestic airlines. For instance, in the US, the foreign ownership of voting shares in airline companies is limited to 25% of the total.

There also exist other effects of ATI that may reduce total welfare and harm consumers. According to Stuart (2006), the agreements have two main negative effects: first, a direct effect which results from a reduction in competition between partners, and second, an indirect effect due to the strategic barriers that alliances may impose on rivals’ entry
decisions. The direct effect is strongly connected to the degree of overlapping routes of partners. Park (1997) distinguishes between alliances with no overlapping routes prior the agreement (complementary alliances) and the scenario where partners compete on some of the routes (parallel alliances). His results suggest that alliances between companies with a high degree of overlapped routes raise airlines profits but reduce consumer welfare. The second result mentioned by Stuart (2006) is linked to airport dominance. The larger the presence of allied carriers in one airport, the more difficult it is for rivals to operate at such an airport. As a result, carriers can deter entry of non-allied airlines and discriminate on interline access to benefit allied partners. A good example is the strategy followed by Air France (AF) at Charles De Gaule Airport (CDG). In 2004, Continental Airlines (CO) allied with AF through the “Skyteam Alliance”. Reitzes and Moss (2008) noticed that after the agreement, CO increased its connecting passengers beyond CDG. At the same time, the non-allied Northwest (NW) and United Airlines (UA) experienced a decrement in the number of passengers originated in the US and terminating beyond CDG.

There exists important theoretical work on alliances. Zhang and Czerny (2012) present an excellent review of recent research about the topic. Close to our work, the aforementioned paper by Brueckner and Whalen (2000) analyzes the effects of alliances on fares. However, their model does not endogenize the decision regarding the formation of alliances. In this regard, Bilotkach (2005) addresses the carriers’ incentives to ally. While in his setting products offered by airlines are vertically differentiated by number of stops, we capture horizontal differentiation (brand-loyalty of travelers). His model also allows for competition between members of the alliance prior the agreement (our model does not). However, his complex setting does not capture economies of traffic density, which are important in the airline industry. In a more general setting, Zhang and Zhang (2006) examine rivalry between alliances where the profit function of each member depends on its own profit and a share of its partner’s profit. Despite this feature, their model does not explicitly introduce the aforementioned specificities of the airline industry.

The closest works to our paper are Flores-Fillol and Moner-Colonques (2007), and Jiang, Wan, and D’Alfonso (2014). Flores-Fillol and Moner-Colonques (2007) present an interesting exercise where carriers operating as monopolists on their respective domestic markets may not find it desirable to ally even if there exist economies of traffic density, product differentiation, and market size effects. There are significant differences with respect to our study: first, Flores-Fillol and Moner-Colonques (2007) propose a two stage-game in which carriers first simultaneously decide if they want to cooperate, and then, in a second stage, set fares. In our work, the decision regarding the alliance formation is made sequentially:
first, the leader airline decides if it wants to form an international alliance. Then, in a second stage, the follower observes the action taken by the leader and decides according to its best response function. Our approach is more consistent with the formation of international alliances connecting the US and Europe. On the other hand, the recent Transpacific agreements between United Airlines (UA) and All Nippon Airways (NH), and American Airlines (AA) and Japan Airlines (JL) can be seen as a result of a simultaneous game.  

Second, Flores-Fillol and Moner-Colonques (2007) assume that fares for international interline trips are just the sum of the nonstop segment fares, whereas we allow interline trips to have individual prices in a more-realistic fashion. Third, in their model, allied airlines jointly maximize profits from domestic and international markets. In our paper, partners only share revenues from international markets, while independently deciding on domestic fares. This is more in line with observed agreements, since airlines are only able to jointly determine fares and share revenues in markets where competition authorities grant ATI (e.g. Delta (DL) and Virgin Australia (VA) on flights connecting the US, Australia and New Zealand, or American Airlines (AA), British Airways (BA), and Iberia (IB) on routes between the US and Europe). Four, in their model, competition only arises in international markets, which include trips with two segments operated by different carriers. Our work, in this regard, is more general, since we assume that carriers compete in both types of markets: domestic and international. Lastly, one of the distinctive features of our model is its ability to capture efficiency gains (losses) as a result of alliance formation.

Finally, Jiang, Wan, and D’Alfonso (2014) develop a theoretical model of cooperation between domestic carriers and global alliances, in which domestic airlines are able to choose which alliance to partner with. Their model includes overlapping routes between members and introduces differences between domestic airlines and global alliances. Our model uses a much simpler network but allows for differences between domestic airlines and, as previously stated, variation in fixed costs as a result of synergies. Another important difference is that while our results clearly depend on the parameter linked to economies of traffic density, Jiang et al (2014) assume, instead, constant marginal costs.  

Finally, their paper does not study the welfare implications of the equilibrium outcomes. As a result, policy suggestions cannot be made as we do.

The structure of the paper is as follows: section 2 introduces the model. Section 3 analyzes the airlines’ profit maximization problem. Section 4 describes the subgame perfect Nash equilibria. Section 5 presents a welfare analysis. Finally, section 6 concludes.

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1 Both immunized alliances were granted on April 2011.

2 They claim that including economies of traffic density does not have a qualitative impact on their results.
2. Model

2.1. **Network and Timing of the Game.** Similar to the network presented by Brueckner and Whalen (2000), we consider two countries (M and V) with two spokes each sharing the same hub airport (H) (Figure 1). In each market there are two competing airlines: 1 and 2 in country M, and 3 and 4 in country V. Carriers 1 and 2 fly in country M connecting the cities A, B and H. On the other hand, carriers 3 and 4 operate in country V offering flights between the cities C, D and H. There are no direct connections for city-pairs AB and CD. Passengers willing to travel between these cities necessarily have to stop at H. Similarly, passengers traveling from one country to the other have to stop at H and change carrier (interline connection). Whereas the model assumes a unique hub for all companies, it would be more realistic to introduce two international hubs, one in each country. However, introducing a network with a hub-to-hub route would make the model more complex since we would have a route where carriers from different countries compete against each other. As a result, our model only captures the effects of complementary alliances, alliances between airlines with no overlapping routes prior to the agreement.

Figure 2 shows the sequence of the game: airline 1 first decides if it wants to establish an alliance with one of the airlines operating in country V (carrier 3), and consequently ask for ATI. Then, carrier 2 observes the decision taken by its competitor and chooses to ally, or not, with airline 4. Finally, airlines simultaneously decide on fares.

In this setting, airlines compete in both domestic and international markets. In this regard, the model is more general than the one proposed by Flores-Fillol and Moner-Colonques (2007), in which carriers only compete for international travelers.

2.2. **Demand Function and Prices.** Following Brueckner and Flores-Fillol (2007) and Brueckner and Whalen (2000), we assume that the utility of individuals from using airline 1 \( u^{(1)} \) is given by

\[
 u^{(1)} = b - p^{(1)} + z
\]

(1)

where \( b \) is the gain from air travel, \( p^{(1)} \) is the ticket price, and \( z \) captures the heterogeneous preference of travelers for carrier 1 relative to carrier 2, which may be positive or negative (there is an analogous utility expression for carrier 3). On the other hand, the utility of individuals from using airline 2 \( u^{(2)} \) is similar but with \( z = 0 \) (analogously for carrier 4). That is,
Travelers will choose airline 1 as long as the ticket price \( p^{(1)} \) is not higher than the one offered by competitor 2 \( p^{(2)} \) plus the constant \( z \) (i.e. \( p^{(1)} < p^{(2)} + z \)). If we assume that \( z \) is uniformly distributed over the interval \( [-\frac{\delta}{2}, \frac{\delta}{2}] \), then we can derive the traffic for carrier 1 as follows:

\[
q^{(1)} = \frac{1}{\delta} \int_{p^{(1)}-p^{(2)}}^{\frac{\delta}{2}} dz = \frac{1}{2} - \frac{p^{(1)} - p^{(2)}}{\delta}
\]

where we assume that \( 1 \geq \delta > 0 \) in order to guarantee positive prices in the optimality conditions. \( \delta \) can be seen as a brand loyalty parameter which captures the effort of carriers to keep customers using, for instance, frequent flyer programs. The higher is \( \delta \), the greater is the dispersion of consumer preferences for particular carriers. We assume that total demand equals 1. As a result, the demand for carrier 2 is equal to

\[
q^{(2)} = 1 - q^{(1)}
\]

We also assume that the gain from air traveling \( (b) \) in (1) and (2) is high enough to avoid the existence of an outside option. That is, travelers always find it preferable to fly rather than using other modes of transportation or to stay at home. This type of demand fits reasonably well for business customers that must travel between cities located far away. In such a case, flying is the most attractive option.

The domestic traffic that company \( i \) faces for connecting the city-pair \( (l, m) \) is denoted by \( q^{(i)}_{lm} \) where \( i \in \{1, 2\}, \ l \in \{A,B\} \) and \( m \in \{A,B,H\} \). For example, airline 1 traffic between the cities \( A \) and \( H \) is denoted \( q^{(1)}_{AH} \). Similarly, \( q^{(ij)}_{ln} \) is the demand for interline flights connecting city-pairs located in different countries, where \( i \in \{1, 2\}, \ j = 3 \) if \( i = 1, \ j = 4 \) if \( i = 2 \), and \( n \in \{C,D\} \). That means that international travelers using carrier 1 in market \( M \) use carrier 3 in market \( V \) (similar reasoning for carriers 2 and 4). For instance, \( q^{(13)}_{AC} \) is the demand that carriers 1 and 3 face for traveling between cities \( A \) and \( C \). In such a case, carrier 1 flies between \( A \) and \( H \), and carrier 3 operates in the spoke \( HC \) (see Figure 1).³

Regarding the network, we assume that all spokes have the same characteristics. This assumption implies that \( q^{(i)}_{AH} = q^{(i)}_{BH} \equiv q^{(i)}_{XH} \) where \( X \in \{A,B\} \). Analogously for the international demand, \( q^{(ij)}_{AC} = q^{(ij)}_{AD} = q^{(ij)}_{BC} = q^{(ij)}_{BD} \equiv q^{(ij)}_{XY} \) where \( Y \in \{C,D\} \).

³Jiang et al (2014) propose a more flexible model in which carriers are allowed to choose their partner.
Fares for city-pairs located in the domestic market are denoted by \( P_{AH}^{(i)} = P_{BH}^{(i)} \equiv P_{XH}^{(i)} \) for non-stop markets, and \( P_{AB}^{(1)} \) for the market \( AB \). Given the previously stated assumptions, \( P_{XH}^{(1)} = P_{XH}^{(3)} \) and \( P_{AB}^{(1)} = P_{AB}^{(3)} \) (similar reasoning applies for carriers 2 and 4). The price scheme for trips between city-pairs located in different countries changes depending on the existence of alliances. \( P_{XY}^{(ij)} \) denotes the interline fare when carriers are allied. On the other hand, if companies do not coordinate, the interline fare will be the sum of the subfares offered by each carrier for traveling between the city located in its country and the hub H. Let \( S_{XY}^{(i)} \) denote the subfare offered by \( i \) for the trip segment \( XH \), and \( S_{XY}^{(j)} \) the subfare set by \( j \) for the spoke \( HY \), where \( i \in \{1, 2\} \), \( j = 3 \) if \( i = 1 \), \( j = 4 \) if \( i = 2 \). In such a case, the final price of the international trip is the sum of the corresponding subfares offered by the two carriers (\( S_{XY}^{(i)} + S_{XY}^{(j)} \)). For example, the joint price offered by airlines 1 and 3 for connecting A and C is \( P_{13}^{AC} \). On the other hand, the price paid by international travelers would be the sum of the two subfares if both carriers do not ally (\( S_{AC}^{(1)} + S_{AC}^{(3)} \)). Given the structure of the network and carriers, the ticket price offered by \( i \) and \( j \) will be the same for any international route. That is, \( P_{AC}^{(ij)} = P_{BC}^{(ij)} = P_{AD}^{(ij)} = P_{BD}^{(ij)} \equiv P_{XY}^{(ij)} \) if \( i \) and \( j \) cooperate, and \( S_{XY}^{(i)} = S_{XY}^{(j)} \) if carriers \( i \) and \( j \) do not ally.

2.3. Cost Function. The cost function captures two effects present in the airline industry: economies of traffic density and synergies derived from the alliance. That is, the marginal cost is decreasing with demand (economies of traffic density), and there may be efficiency gains, or losses, derived from the alliance (synergies). As we previously stated, previous literature has neglected changes in fixed costs due to the existence of alliance synergies.

The spoke cost function of carrier \( i \) is similar to the one appearing in Brueckner and Whalen (2000) except for the existence of synergies as a result of the alliance, and it is given by

\[
(5) \quad \epsilon^{(i)}(Q^{(i)}) = Q^{(i)} - \frac{1}{2} \theta Q^{(i)2} - \frac{F^{(i)}}{2} \mathbb{1}^{(i)}\{\text{Alliance}\} + \frac{G}{2}
\]

where \( Q^{(i)} \) corresponds to the spoke traffic of carrier \( i \). \( \theta \) is interpreted as the strength of the economies of traffic density: the higher the value of \( \theta > 0 \), the lower is the cost of an additional passenger. The constant \( F^{(i)} \) captures the possible synergies (positive or negative) as a result of the alliance, and it is multiplied by the indicator function \( \mathbb{1}\{\cdot\} \) which takes the value 1 if airlines \( i \) and \( j \) coordinate and zero otherwise. If \( F^{(i)} > 0 \), airline cooperation reduces the fixed costs of allied carriers as a result of gains in efficiency (e.g. more efficient scheduling or integration of ground activities). On the other hand, \( F^{(i)} < 0 \) leads to an
increase in fixed costs. It can be interpreted as a cost of coordination, and it may arise when allied companies are too asymmetric. We assume that these synergies are not necessarily equal for carriers 1 and 2, but we impose that $F^{(1)} = F^{(3)}$ and $F^{(2)} = F^{(4)}$. Finally, $G$ denotes other fixed costs, which are assumed to be the same for all carriers.

As previously noted, $Q^{(i)}$ equals total demand (domestic and international) using one of the spokes in country $M$ and carrier $i$, and is given by

\begin{equation}
Q^{(i)} = q^{(i)}_{XH} + q^{(i)}_{AB} + 2q^{(ij)}_{XY}
\end{equation}

where $q^{(i)}_{XH}$ is the demand for carrier $i$ in city-pair $XH$, $q^{(i)}_{AB}$ is the domestic demand for the market $AB$, whose passengers necessarily stop at the hub $H$ to reach destination, and $q^{(ij)}_{XY}$ is the international traffic in city-pair $XY$ and using carriers $i$ and $j$ in the spokes $XH$ and $HY$ respectively. Note that the term $q^{(ij)}_{XY}$ is multiplied by 2 because any spoke in country $M$ is fed by demand coming from the spokes $HC$ and $HD$ located in country $V$.

### 3. Solving the Model

Airlines are assumed to be profit maximizing firms with respect to fares, and they decide if it is profitable to establish an international alliance, and consequently ask for ATI from the competition authorities. We assume that airline 1 decides first (leader). Afterwards, airline 2 (follower) observes the action taken by the leader and decides according to its best response function. Finally, airlines simultaneously decide on prices. The equilibrium concept used to solve the model is the subgame perfect Nash equilibrium. This type of model is solved backwards, we first find the optimal fares and profits for each of the possible alliance scenarios, then we solve for the follower (carrier 2) taking as given the decision of the leader (carrier 1), and finally solve for carrier 1’s decision knowing the response of the follower.

This section characterizes the optimal ticket prices and profits for each of the possible strategy profiles of carriers. First, we study the case where airlines 1 and 3 do not ally, and airline 2 decides if it wants to reach an agreement with airline 4. Afterwards, we repeat the same analysis where airlines 1 and 3 find it profitable to cooperate offering a joint price for international city-pairs.

As we previously mentioned, the model is similar to the one presented by Brueckner and Whalen (2000) except for the existence of synergies ($F^{(i)}$). Given the assumptions on $F^{(i)}$, both papers lead to the same optimal fares for each of the possible scenarios.
3.1. **Airlines 1 and 3: Non-Alliance.** Assuming that airline 1 is not interested in a deal with airline 3, carrier 2 compares the profit that obtains depending on its decision (ally vs not to ally).

3.1.1. **Airlines 2 and 4: Non-Alliance.** In this case, no one makes a deal. Hence, there are no joint fares for trips connecting international city-pairs. Airline 1 maximizes its profits with respect to prices in the domestic market \((P^{(1)}_{XH}, P^{(1)}_{AB})\), and with respect to the subfare \(S^{(1)}_{XY}\) for connecting cities located in different countries without taking into account the effects of its decision on airline 3 (similar reasoning for carrier 2).

In this scenario, we just need to solve the maximization problem of one airline to get the profits and fares offered by the others. Note that the only element differentiating carriers \((F^{(i)})\) only appears when there exist alliances.

The revenue function for carrier \(i\) is

\[
R^{(i)} = 2P^{(i)}_{XH}q^{(i)}_{XH} + P^{(i)}_{AB}q^{(i)}_{AB} + 4S^{(i)}_{XY}q^{(ij)}_{XY}
\]

where \(i \in \{1, 2\}, j = 3\) if \(i = 1\), and \(j = 4\) if \(i = 2\).

The sum of revenues from operating the spokes \(AH\) and \(BH\) is equal to \(2P^{(i)}_{XH}q^{(i)}_{XH}\), the revenues from the city-pair \(AB\) are \(P^{(i)}_{AB}q^{(i)}_{AB}\), and the revenues from international routes are \(4S^{(i)}_{XY}q^{(ij)}_{XY}\).

As we already noted, \(q^{(i)}_{lm}\) depends negatively on the price offered by carrier \(i\), and it is positively correlated with the price offered by its competitor. Using (3), the non-stop traffic for the domestic city-pair \(XH\) is

\[
q^{(i)}_{XH} = \frac{1}{2} - \frac{(P^{(i)}_{XH} - P^{(i')}_{XH})}{\delta}
\]

where \(i' \in \{1, 2\}\) and \(i' \neq i\). Similarly, the domestic demand for the city-pair \(AB\) is equal to

\[
q^{(i)}_{AB} = \frac{1}{2} - \frac{(P^{(i)}_{AB} - P^{(i')}_{AB})}{\delta}
\]

and the demand for the international route \(XY\) is

\[
q^{(ij)}_{XY} = \frac{1}{2} - \frac{(S^{(i)}_{XY} + S^{(j)}_{XY}) - (S^{(i')}_{XY} + S^{(j')}_{XY})}{\delta}
\]

where \(i' \in \{1, 2\}, j' \in \{3, 4\}, i' \neq i, j' \neq j, j = 3\) if \(i = 1\), and \(j = 4\) if \(i = 2\).
Using the revenue function (7) and the spoke cost function (5), carrier $i$ decides fares according to the following maximization problem:

$$\max_{P^{(i)}_{XH}, P^{(i)}_{XY}, P^{(i)}_{AB}} R^{(i)}(Q^{(i)}) - 2c(Q^{(i)})$$

(11)

Remember that the cost function $c(Q^{(i)})$ refers to a single spoke, and carriers are assumed to operate on two spokes.

Given the maximization problem, the corresponding optimal fare for trips between the cities $X$ and $H$ is

$$P^{(i)}_{XH_{na1na2}} = \frac{1}{2}(2 + \delta - 4\theta)$$

(12)

Note that the sub-index of the optimal fare does not only denote markets but also the strategy profile of carriers: $a1$ means that carrier 1 allies, and $na1$ denotes carrier 1 does not cooperate with carrier 3 (analogously for carrier 2).

Similarly, the optimal fare for domestic flights connecting $A$ and $B$ is

$$P^{(i)}_{AB_{na1na2}} = \frac{1}{2}(4 + \delta - 8\theta)$$

(13)

Remember that when airlines do not cooperate, the interline fare for the international trip is the sum of the two subfares offered by the corresponding carrier in each country. In this scenario, the optimal subfare equals

$$S^{(i)}_{XY_{na1na2}} = \frac{1}{2}(2 + \delta - 4\theta)$$

(14)

The solution is the same for both carriers since both use the same strategy (not to ally).

Given the optimal fares, the corresponding profits are

$$\Pi^{(i)}_{na1na2} = \frac{7\delta}{4} - 4\theta - G$$

(15)

We observe that prices and profits depend on $\delta$ and $\theta$. Such exogenous parameters will determine the responses of airlines. As we previously stated, $\delta$ captures the dispersion of preferences for particular airlines. The higher the value of $\delta$, the lower is the probability that a traveler switches carrier. That explains why profits and prices are positively correlated with the preference parameter. On the other hand, there is a negative correlation of prices and profits with respect to the parameter linked to economies of traffic density ($\theta$). This
relationship is explained by the competitive environment of the sector. The higher the value of \( \theta \), the lower is the cost of providing services. Since carriers are competing, that creates downward pressure on prices and, consequently, on profits.

3.1.2. Airlines 2 and 4: Alliance. Assume now the asymmetric case where airlines 1 and 3 do not ally, and airlines 2 and 4 decide to cooperate in international markets. In this case, carriers 2 and 4 (followers) offer a joint price for connecting cities located in different countries (\( P_{XY}^{(24)} \)).

The revenue function of carrier 1 has the same structure as the previous case (Equation 7), but the revenue function for carrier 2 is

\[
R^{(2)} = 2P_{XH}^{(2)} q_{XH}^{(2)} + P_{AB}^{(2)} q_{AB}^{(2)} + 2P_{XY}^{(24)} q_{XY}^{(24)}
\]

and the demand for international markets is given by

\[
q_{XY}^{(13)} = \frac{1}{2} - \frac{(S_{XY}^{(1)} + S_{XY}^{(3)}) - P_{XY}^{(24)}}{\delta}
\]

\[
q_{XY}^{(24)} = \frac{1}{2} - \frac{P_{XY}^{(24)} - (S_{XY}^{(1)} + S_{XY}^{(3)})}{\delta}
\]

Note that the only part of the revenues shared by cooperating carriers is the one generated by the international routes, not the domestic ones.\(^4\) For simplicity, we assume that carriers do not have bargaining power and split the revenues from interline trips in the same proportion. That explains why the last term on the right hand side in (16) is multiplied by 2.

Given the revenue and cost functions, carriers solve the following maximization problems. For carrier 1,

\[
\max_{P_{XH}^{(1)}, S_{XY}^{(1)}, P_{AB}^{(1)}} R^{(1)} - 2c(Q^{(1)})
\]

Carrier 2 solves a similar expression, but fares for international trips are jointly set with carrier 4,

\[
\max_{P_{XH}^{(2)}, P_{XY}^{(2)}, P_{AB}^{(2)}} R^{(2)} - 2c(Q^{(2)})
\]

\(^4\)Other papers use a less realistic approach assuming that partners share both domestic and international revenues (e.g. Flores-Fillol and Moner-Colonques (2007)).
Given the optimization problem, the corresponding optimal domestic fares for carrier 1 are

\[ P_{XHna1a2}^{(1)} = 1 + \frac{23\delta}{48} + \frac{\delta^2}{48(\delta - 4\theta)} - 2\theta \]  

(21)

\[ P_{ABna1a2}^{(1)} = \frac{3\delta(4 + \delta) - (48 + 35\delta)\theta + 96\theta^2}{6(\delta - 4\theta)} \]  

(22)

and the optimal subfare charged for the spoke operated by carrier 1 in the corresponding international trip is

\[ S_{XYna1a2}^{(1)} = \frac{1}{8}(8 + 3\delta - 16\theta) \]  

(23)

As previously noted, one of the limitations of the model presented by Flores-Fillol and Moner-Colonques (2007) is the assumption that fares for international interline trips equal the sum of the nonstop segment fares. Our model is more realistic, since we allow interline trips to have different prices (i.e. \( S_{XYna1a2}^{(1)} \neq P_{XHna1a2}^{(1)} \)).

For carrier 2, the optimal ticket prices are given by

\[ P_{XHna1a2}^{(2)} = 1 + \frac{\delta(6\delta - 25\theta)}{12(\delta - 4\theta)} - 2\theta \]  

(24)

\[ P_{ABna1a2}^{(2)} = 2 + \frac{\delta(3\delta - 13\theta)}{6(\delta - 4\theta)} - 4\theta \]  

(25)

\[ P_{XYna1a2}^{(24)} = 2 + \frac{\delta(5\delta - 22\theta)}{8(\delta - 4\theta)} - 4\theta \]  

(26)

Note that optimal fares do not depend on the synergies generated from the alliance \( F^{(i)} \).

Given fares, the corresponding profits are

\[ \Pi_{na1a2}^{(1)} = \frac{63\delta^2 - 719\delta^2\theta + 2644\delta\theta^2 - 3072\theta^3}{48(\delta - 4\theta)^2} - G \]  

(27)

\[ \Pi_{na1a2}^{(2)} = \frac{147\delta^2 - 1538\delta^2\theta + 5340\delta\theta^2 - 6144\theta^3}{96(\delta - 4\theta)^2} + F^{(2)} - G \]  

(28)
where, as we previously noted, $F^{(2)}$ captures possible synergies generated by the alliance between carriers 2 and 4.

3.2. **Airlines 1 and 3: Alliance.** In this section, we analyze the optimal fares and profits when airlines 1 and 3 ask for antitrust immunity.

3.2.1. **Airlines 2 and 4: Non-Alliance.** Consider the case where airline 1 agrees with carrier 3, and airlines 2 and 4 do not cooperate. This case is already solved above (section 3.1.2). By symmetry, we just need to exchange carrier 1 for carrier 2 and vice versa. Consequently, the optimal fares for carrier 1 are

\[
P_{XYa1na2}^{(13)} = 2 + \frac{5\delta - 22\theta}{8(\delta - 4\theta)} - 4\theta
\]

\[
P_{XHa1na2}^{(1)} = 1 + \frac{6\delta - 25\theta}{12(\delta - 4\theta)} - 2\theta
\]

\[
P_{AHa1na2}^{(1)} = 2 + \frac{3\delta - 13\theta}{6(\delta - 4\theta)} - 4\theta
\]

and the optimal decisions for carrier 2 with respect to ticket prices are

\[
S_{XYa1na2}^{(2)} = \frac{1}{8}(8 + 3\delta - 16\theta)
\]

\[
P_{XHa1na2}^{(2)} = 1 + \frac{23\delta}{48} + \frac{\delta^2}{48(\delta - 4\theta)} - 2\theta
\]

\[
P_{AHa1na2}^{(2)} = \frac{3\delta(4 + \delta) - (48 + 35\delta)\theta + 96\theta^2}{6(\delta - 4\theta)}
\]

Finally, the corresponding profits are given by

\[
\Pi_{a1na2}^{(1)} = \frac{147\delta^2 - 1538\delta^2\theta + 5340\delta\theta^2 - 6144\theta^3}{96(\delta - 4\theta)^2} + F^{(1)} - G
\]

\[
\Pi_{a1na2}^{(2)} = \frac{63\delta^2 - 719\delta^2\theta + 2644\delta\theta^2 - 3072\theta^3}{48(\delta - 4\theta)^2} - G
\]
3.2.2. *Airline 2 and 4: Alliance.* The last case is when everybody joins an alliance. Both airlines in country $M$ set a joint fare with their respective partner in country $V$ ($P_{XY}^{(13)}$ and $P_{XY}^{(24)}$). Then, the revenue function is equal to

$$R^{(i)} = 2P_{XY}^{(i)}q_{XY}^{(i)} + P_{AB}^{(i)}q_{AB}^{(i)} + 2P_{XY}^{(ij)}q_{XY}^{(ij)}$$

for $i \in \{1, 2\}$, $j = 3$ if $i = 1$ and $j = 4$ if $i = 2$.

Under this scenario, carriers set fares according to the following maximization problem:

$$\max_{P_{XY}^{(i)}, P_{XY}^{(ij)}, P_{AB}^{(i)}} R^{(i)} - 2c(Q^{(i)})$$

The solution of the problem is

$$P_{XY}^{(13)} = P_{XY}^{(24)} = \frac{1}{2}(4 + \delta - 8\theta)$$

$$P_{AB}^{(1)} = P_{AB}^{(2)} = \frac{1}{2}(4 + \delta - 8\theta)$$

$$P_{XY}^{(1)} = P_{XY}^{(2)} = \frac{1}{2}(2 + \delta - 4\theta)$$

and the corresponding profits are

$$\Pi_{a1a2}^{(1)} = \frac{5\delta}{4} - 4\theta + F^{(1)} - G$$

$$\Pi_{a1a2}^{(2)} = \frac{5\delta}{4} - 4\theta + F^{(2)} - G$$

By construction, the maximization problems are well defined (profit function concave and positive prices) for values $1 \geq \delta > 4\theta > 0$. Appendix 1 shows further details about the necessary conditions on $\theta$ and $\delta$ to guarantee strict concavity of the profit functions.

3.3. *Price Behavior.* Following Brueckner and Whalen (2000), we compare the fares for each of the discussed scenarios and observe the following relationships:

$$P_{XY}^{(i)} < P_{XY}^{(13)} < S_{XY}^{(2)} < S_{XY}^{(4)} < S_{XY}^{(i)}$$

$$P_{AB}^{(i)} < P_{AB}^{(i)} = P_{AB}^{(i)} < P_{AB}^{(2)}$$
For international city-pairs, the interline fare when there are no alliances is higher than in any other scenario (Equation 43). In this case, each airline maximizes profits without taking into account the corresponding foreign carrier. As a result, double markups lead to higher prices. On the other hand, prices when both carriers are allied are lower because airlines internalize the partner profits, eliminating double marginalization. In between, we have prices in asymmetric scenarios where one of the airlines maximizes the joint international revenues with its partner, and its competitor does not.

As we previously noted, the extent of the alliances is not only limited to international routes, but also affects the behavior of airlines in domestic markets. We observe that prices in the domestic market do not change when the strategy of carriers is the same (see Equations 44 and 45). This is a direct consequence of the assumption on demand (Equation 3). By construction, total demand is always equal to 1 and does not depend on fares. However, when the decisions of players are different, the one which decides to ally in the international market is able to take advantage of economies of traffic density and offer a lower fare than its competitor. We already saw that when a carrier cooperates, it offers lower fares on the international routes. That leads to a reduction in marginal costs via incremental passengers, making profitable lower prices in the domestic market. On the other hand, the carrier which decides to operate on its own has an increment to marginal costs due to the business stealing effect by its competitor. Consequently, it has to raise prices in both international and domestic markets. However, the brand loyalty of travelers prevents all passengers from changing carrier.

4. SUBGAME PERFECT NASH EQUILIBRIA

In this section we study the set of subgame perfect Nash equilibria of the game. As we previously mentioned, the leader first decides to ally knowing the response of carrier 2. Afterwards, the follower chooses. This leader-follower structure is clearly different from the simultaneous-move assumption of other papers (e.g. Flores-Fillol and Moner-Colonques (2007)).
The model is solved backwards: first, knowing the optimal fares and profits for each of the possible scenarios, we analyze the best response of airline 2 (follower) taking as given the decision of carrier 1. Then, we study the equilibrium behavior of carrier 1 knowing the reaction of carrier 2.

We characterize the set of equilibria as a function of the brand loyalty of travelers \((\delta)\), economies of traffic density \((\theta)\), and synergies derived from the agreement \((F^{(i)})\). For comparative reasons, we assume that the fixed costs of carriers 2 and 4 are not a function of their level of cooperation \((F^{(2)} = F^{(4)}=0)\). In line with previous results (e.g. Flores-Fillol and Moner-Colonques (2007) and Jiang et al (2014)), we will see that creating an alliance is not always the best carrier’s strategy given the response of the competitor.

4.1. 2nd Stage of the Game: Best Response of Carrier 2 (follower). We solve the second stage of the game and characterize the best response function of the follower (carrier 2) given the decision of the leader (carrier 1).

If carriers 1 and 3 do not cooperate, we ask what is the best response of carrier 2 as a function of the strength of the brand loyalty of travelers \((\delta)\) and the parameter linked to economies of traffic density \((\theta)\).

The solid line in Figure 3 shows the function corresponding to the \((\delta,\theta)\) pair of values for which carrier 2 is indifferent between being allied or not \((\Pi_{na1a2}^{(2)} = \Pi_{na1na2}^{(2)})\). The area between the dotted line and the x-axis represents the set of admissible \((\delta,\theta)\) pairs.\(^5\) The shaded area shows the \((\delta,\theta)\) pairs for which airline 2 prefers to follow the same strategy as carrier 1 \((\Pi_{na1a2}^{(2)} < \Pi_{na1na2}^{(2)})\). That is, both are not interested in creating an alliance. For other admissible values of \((\delta,\theta)\), making a deal is profitable for carrier 2 \((\Pi_{na1a2}^{(2)} > \Pi_{na1na2}^{(2)})\).

Figure 3 can be interpreted as follows: given the decision of carrier 1 of not asking for ATI, airline 2 only finds it profitable to reach an agreement if economies of traffic density \((\theta)\) are big enough to compensate for the brand-loyalty effect \((\delta)\). Otherwise, the airline would not find it interesting to share its profits from international travelers with carrier 4. Remember that we assume that there are no synergies as a result of an alliance between carriers 2 and 4 \((F^{(2)} = F^{(4)} = 0)\). In the case where they benefit from synergies \((F^{(2)} = F^{(4)} > 0)\), the solid line would shift down, reducing the set of possible values of \(\delta\) and \(\theta\) for which carriers are not interested in cooperating (other way around for \(F^{(2)} = F^{(4)} < 0)\).

If we assume airlines 1 and 3 cooperate, the solid lines in Figure 4 show the \((\delta,\theta)\) pairs for which airline 2 finds an agreement with carrier 4 as profitable as operating on its

\(^5\)As previously mentioned, the model is well defined when the hessian of the profit functions with respect to fares is negative definite and fares are positive. Conditions that are satisfied for \(1 \geq \delta > 4\theta > 0\).
own ($\Pi_{a_{1a2}}^{(2)} = \Pi_{a_{1na2}}^{(2)}$). The shaded area represents the $(\delta, \theta)$ pairs for which airline 2’s best response is to coordinate. For other admissible values of $(\delta, \theta)$, the airline 2 is not interested in sharing its international revenues with carrier 4. In this scenario, having positive synergies ($F(2) = F(4) > 0$) would shift the profit function $\Pi_{a_{1a2}}^{(2)}$, increasing the set of $(\delta, \theta)$ pairs for which carrier 2 is interested in cooperating (other way around for $F(2) = F(4) < 0$).

If we merge Figures 3 and 4, we can illustrate the best response of airline 2 as a function of the parameters $(\delta, \theta)$ and the decision of carrier 1 (Figure 5). The area between the dotted line and the x-axis represents the set of admissible $(\delta, \theta)$ pairs. The dashed lines limit the $(\delta, \theta)$ pairs for which carrier 2 prefers to ally when airline 1 also allies. The area between the continuous line and the x-axis represents the $(\delta, \theta)$ pairs for which carrier 2’s best response is not to cooperate with carrier 4 if carrier 1 decides not to ally. Finally, the shaded area in Figure 5 shows the $(\delta, \theta)$ pairs for which whatever carrier 1 chooses, the airline 2 is not interested in a deal with airline 4. That is, it is the set $(\delta, \theta)$ for which the dominant strategy for carrier 2 is not to cooperate with carrier 4.

4.2. 1st Stage of the Game: Best Response of Carrier 1 (leader). Remember the sequence of the game: given the parameter that captures the brand-loyalty of travelers ($\delta$), the parameter linked to economies of traffic density ($\theta$), and the possible synergies ($F(1)$), the leader (carrier 1) decides to ally, or not, with carrier 3. Then, carrier 2 observes the decision taken by the leader and decides if it follows the same strategy with airline 4. Finally, airlines simultaneously decide on fares.

Assuming that $(\delta, \theta)$ pair is located in the shaded area in Figure 5 and knowing the best response of carrier 2, it is up to carrier 1 to decide. Airline 1 will ally if $\Pi_{a_{1a2}}^{(1)} \geq \Pi_{a_{1na2}}^{(1)}$. Given $\delta$ and $\theta$, the decision clearly depends on the value of $F(1)$. Comparing the profits that carrier 1 obtains if it coordinates (Equation 35) with the ones obtained operating by its own (Equation 15), the carrier 1’s best response knowing the reaction of airline 2 is to ally if

$$F(1) \geq \frac{21\delta^3 - 190\delta^2\theta + 420\delta\theta^2}{96(\delta - 4\theta)^2} > 0$$

The term in the middle in (46) is strictly positive.\(^6\) That means that carrier 1 has to benefit from a reduction in its fixed cost in order to find an ATI profitable.

Another case is depicted in Figure 6. The shaded area shows the $(\delta, \theta)$ pairs for which airline 2 mimics the carrier 1’s decision, that is, to ally if carrier 1 allies and not to ally if

\(^6\)We plotted the middle term in (46) and saw that the value function is greater than zero for any $(\delta, \theta)$ pair satisfying the constraint on parameters ($1 \geq \delta > 4\theta > 0$).
carrier 1 does not. In this scenario, carriers 1 and 3 cooperate if $\Pi_{a1a2}^{(1)} \geq \Pi_{na1a2}^{(1)}$. If we compare the expressions (41) and (15), carriers 1 and 3 are interested in applying for ATI if

$$F^{(1)} \geq \frac{\delta}{2}$$

Again, some positive synergies are necessary in order for carriers 1 and 3 to decide to coordinate.

The shaded area in Figure 7 represents the pairs $(\delta, \theta)$ for which whatever the firm 1 chooses, the airline 2 is always interested in a deal with airline 4. That is, to cooperate is a dominant strategy for carrier 2. In this situation, the carrier 1’s best response to carrier 2’s decision is to ally if $\Pi_{a1a2}^{(1)} \geq \Pi_{na1a2}^{(1)}$. Using expressions (41) and (27),

$$F^{(1)} \geq \frac{\delta(3\delta^2 - 47\delta\theta + 148\theta^2)}{48(\delta - 4\theta)^2}$$

The right hand side in (48) is negative.\(^7\) That means that carrier 1 is interested in coordinating even if the accompanied synergies are negative. Given the response of carrier 2, airline 1’s profit would be lower operating alone. As a result, airline 1 accepts some degree of cost inefficiency due to the agreement.

Finally, the shaded area in Figure 8 shows the pairs $(\delta, \theta)$ for which airline 2’s best response is to choose the opposite strategy followed by carrier 1: if carrier 1’s strategy is to ally, carrier 2 prefers not to coordinate with carrier 4 and vice versa. In this scenario, carriers 1 and 3 cooperate if $\Pi_{a1a2}^{(1)} \geq \Pi_{na1a2}^{(1)}$. This condition holds for

$$F^{(1)} \geq -\delta(21\delta^2 - 100\delta\theta + 52\theta^2)$$

The right hand side in (49) is negative.\(^8\) That means that even if the alliance implies negative synergies for carrier 1, it may be in its best interest to coordinate with carrier 3 to deter the alliance formation between carriers 2 and 4. As Zhang and Zhang (2006) point out, the threat of coordination by the follower alone can result in a complementary alliance, even if forming an alliance leads to higher fixed costs ($F^{(1)} < 0$).

Figure 9 summarizes the best response of carrier 2 given the strategy followed by carrier 1, and the parameters $\delta$ and $\theta$. Region $A$ denotes the set of pairs $(\delta, \theta)$ for which whatever

\(^7\)We plotted the right hand side in the inequality and saw that the value function is less than zero for any pair $(\delta, \theta)$ satisfying the constraint on parameters ($1 \geq \delta > 4\theta > 0$).

\(^8\)Idem as footnote 3.
airline 1 does, carrier 2 prefers not to ally. If the \((\delta, \theta)\) pair is located in region \(B\), carrier 2 will follow the same strategy adopted by carrier 1. In region \(C\), airline 2 always prefers to ally. Finally, if the \((\delta, \theta)\) pair is in region \(D\), carrier 2’s best response is to choose the opposite strategy followed by carrier 1.

The next proposition summarizes the subgame perfect Nash equilibria of the game:

**Proposition 1:** Given the vector \((\delta, \theta)\) and \(F^{(1)}\), with \(\delta \in (0, 1]\) being the parameter that determines the traveler loyalty to a carrier, \(\theta \in (0, \frac{\delta}{4})\) the parameter that characterizes the economies of traffic density, and \(F^{(1)}\) the possible synergies derived from the alliance:

1. If carrier 2 always finds it more profitable to not ask for antitrust immunity, carrier 1’s best response is to ask for antitrust immunity if \(F^{(1)} \geq \frac{21\delta^3 - 100\delta^2 \theta + 420\delta \theta^2}{96(\delta - 4\theta)^2}\), and operate on its own otherwise.

2. If carrier 2’s best response is to mimic the strategy adopted by carrier 1, carrier 1’s best response is to ask for antitrust immunity if \(F^{(1)} \geq \delta\), and operate on its own otherwise.

3. If carrier 2 always finds it more profitable to ask for antitrust immunity, carrier 1’s best response is to ask for antitrust immunity if \(F^{(1)} \geq \frac{\delta(3\delta^2 - 475\delta + 148\theta^2)}{48(\delta - 4\theta)^2}\), and operate on its own otherwise.

4. If carrier 2’s best response is to follow the opposite to the strategy adopted by carrier 1, carrier 1’s best response is to ask for antitrust immunity if \(F^{(1)} \geq -\frac{\delta(21\delta^2 - 100\delta \theta + 52\theta^2)}{96(\delta - 4\theta)^2}\), and operate on its own otherwise.

### 5. Welfare Analysis

In the previous section we analyzed the subgame perfect Nash equilibria of carriers given the parameters \((\theta, \delta, F^{(1)})\). These strategies are the result of the profit maximization behavior of carriers. However, it is the decision of competition authorities to grant antitrust immunity to the international alliances (ATI).

This section analyzes the decisions of the competition authorities given the strategy followed by carriers. We discuss under which circumstances granting ATI increases total welfare.

We define the welfare of country \(M\) \((W_{s1s2})\) as the total traveler benefits net of the airline costs on the spokes located in the country. After some algebraic manipulations and using expressions (1), (2) and (5), the total welfare is equal to

\[
W_{s1s2} = \int_{\delta}^{\frac{2}{\delta}} [b + z] \frac{1}{\delta} dz + \int_{\frac{\delta}{20}}^{\frac{\delta}{\delta} - p(2)} b \frac{1}{\delta} dz - 2(c^{(1)}_{s1s2} + c^{(2)}_{s1s2})
\]
where \( s1 \in \{a1, na1\} \) and \( s2 \in \{a2, na2\} \) denote the strategies followed by airlines 1 and 2 respectively. Given that both countries \((M, V)\) have similar characteristics, the welfare analysis in country \(M\) is equivalent for country \(V\).

5.1. **Case 1: airline 2 mimics airline 1.** The simplest case is when airline 2 follows the strategy adopted by airline 1 (region \(B\) in Figure 9). In this scenario, the regulator prefers both companies to ally rather than none if \( W_{a1a2} \geq W_{na1na2} \). Using Equation (50) evaluated at the corresponding optimal fares, \( W_{a1a2} \geq W_{na1na2} \) holds for \( F^{(1)} \geq 0 \). Hence, competition authorities should grant ATI to both carriers if the alliance of carriers 1 and 3 generates positive synergies. Remember that in this scenario firms ally if \( F^{(1)} \geq \frac{\delta}{2} \) (see Equation 47). Hence, if the best response of airline 1 is to ally and the best response of airline 2 is to mimic the leader’s strategy, total welfare increases if authorities grant ATI to both carriers.

5.2. **Case 2: airline 2 does not ally whatever airline 1 does.** A different case is when whatever airline 1 does, airline 2 is not interested in a deal with airline 4 (region \(A\) in Figure 9). Given the equilibrium prices found in the previous section, regulators will be interested in having one alliance (airline 1) rather than none if \( W_{a1na2} \geq W_{na1na2} \). After some algebraic manipulation, this condition leads to

\[
F^{(1)} \geq \frac{\delta (255\delta^2 - 2028\delta \theta + 4052\theta^2)}{96(\delta - 4\theta)^2} > 0 \tag{51}
\]

The middle term in (51) is positive for any pair \((\delta, \theta)\) given the constraints on parameters.\(^9\) That means that carrier 1 must benefit from positive synergies in order for the alliance to be interesting from a welfare point of view.

In the previous section we saw that under this scenario, airline 1 is willing to ally if Equation (46) holds. Given the set of feasible values for \(\theta\) and \(\delta\), the lowest bound for \(F^{(1)}\) in (46) is positive but smaller than the lowest bound in the welfare condition (51).\(^10\) Hence, small benefits from synergies can trigger cooperation between airlines 1 and 3. However, we need bigger positive synergies in order to have the agreement welfare improving. In this case, carrier 1 benefits from positive synergies and more travelers. On the other hand carrier 2 loses traffic, decreasing its profits. As a result, we need higher positive synergies in the total welfare condition to compensate carrier 2’s losses.

\(^9\)We plotted the middle term in (51) and saw that the value function is greater than zero for any \((\delta, \theta)\) pair satisfying the constraint on parameters \((1 \geq \delta > 4\theta > 0)\).

\(^{10}\)We plotted both functions and compared them.
Hence, if the best response of airline 1 is to ally, the decision of airline 2 is not to coordinate whatever airline 1 does, and condition (51) holds for the generated synergies, total welfare increases if competition authorities grant ATI to airline 1. If condition (51) is not satisfied, it is better from a total welfare point of view to ban the alliance.

5.3. **Case 3: airline 2 allies whatever airline 1 does.** In this case whatever carrier 1 does, carrier 2 is interested in reaching an agreement with the foreign airline 4 (region $C$ in Figure 9). Again, both alliances would be welfare improving if $W_{a1a2} \geq W_{na1a2}$ and $W_{a1a2} \geq W_{na1na2}$. These two conditions hold for positive synergies ($F^{(1)} \geq 0$).

Remember that in this scenario airline 1 is willing to ally if the generated synergies satisfy (48). The lowest value of $F^{(1)}$ for (48) being satisfied is negative, but the total welfare condition requires positive synergies. As a result, if both carriers’ best response is to ask for antitrust immunity and the generated synergies are positive, then granting ATI increases total welfare.

On the other hand, if carrier 1 is not interested in a deal or the alliance synergies are negative, then competition authorities should be concerned about granting ATI. In this case, total welfare is greater having one alliance (carrier 2) than two. However, we have to see if it is better to grant ATI to carrier 2 than not to do so. This case is already discussed in section (5.2). Since the synergies generated by the alliance of carrier 2 and 4 are assumed to be equal to zero, condition (51) does not hold. Hence, competition authorities should not grant ATI to either carrier 1 or carrier 2.

5.4. **Case 4: airline 2 follows the opposite to the strategy adopted by airline 1.** In this last case, the best response of carrier 2 is doing the opposite of what carrier 1 does (region $D$ in Figure 9). That is, if carrier 1 allies, carrier 2 prefers not to ally and vice versa. In this scenario, the total welfare condition is $W_{a1,na2} \geq W_{na1,a2}$, which is equivalent to $F^{(1)} \geq F^{(2)} = 0$. At the same time, we have to see if it is better to have one alliance than none. We saw in section (5.2) that the only case in which having one alliance is better than none is when condition (51) is satisfied. Hence, in this scenario competition authorities should grant ATI to carrier 1 only if Equation (51) holds for the generated synergies, and none otherwise.

The following proposition summarizes the previous analysis:

**Proposition 2:** Given the vector $(\delta, \theta)$ and $F^{(1)}$, with $\delta \in (0, 1]$ being the parameter that determines the loyalty of travelers to a carrier, $\theta \in (0, \frac{2}{3})$ the parameter that characterizes the economies of traffic density, and $F^{(1)}$ the possible synergies derived from the alliance:
(1) If carrier 2’s best response is to mimic the strategy adopted by carrier 1, and carrier 1’s best response is to ask for antitrust immunity, then granting ATI to both carriers increases total welfare.

(2) If carrier 2 always finds it more profitable to operate on its own, carrier 1’s best response is to ask for antitrust immunity, and \( F^{(1)} \geq \frac{\delta(255\delta^2-2028\delta+4052\theta^2)}{96(\delta-4\theta)^2} \), then granting ATI to carrier 1 increases total welfare. On the other hand, if \( F^{(1)} < \frac{\delta(255\delta^2-2028\delta+4052\theta^2)}{96(\delta-4\theta)^2} \), competition authorities should ban the agreement between carriers 1 and 3.

(3) If carrier 2 always finds it more profitable to ask for antitrust immunity, carrier 1’s best response is also to ask for antitrust immunity and \( F^{(1)} \geq 0 \), then granting ATI to both carriers increases total welfare. On the other hand, if carrier 1 is not interested in a deal or the alliance synergies are negative, then competition authorities should ban both alliances.

(4) If carrier 2’s best response is to follow the opposite to the strategy adopted by carrier 1, carrier 1’s best response is to ask for antitrust immunity, and \( F^{(1)} \geq \frac{\delta(255\delta^2-2028\delta+4052\theta^2)}{96(\delta-4\theta)^2} \), then granting ATI to carrier 1 increases total welfare. On the other hand, if \( F^{(1)} < \frac{\delta(255\delta^2-2028\delta+4052\theta^2)}{96(\delta-4\theta)^2} \), competition authorities should ban any type of agreement between carriers.

6. Conclusion

This paper analyzes the mechanisms that lead airlines to apply for an immunized alliance (ATI) in international markets. ATI removes restrictions in capacity and pricing on international markets operated by the allied carriers.

Our sequential game shows that depending on the brand-loyalty of travelers, economies of traffic density, and the synergies derived from the agreement, carriers might or might not be interested in looking for an international partner. Moreover, the extent of the alliance will not only affect the international markets where airlines coordinate, but also the nature of competition in domestic markets. If economies of traffic density and synergies are not large relative to the brand-loyalty dispersion of travelers, then allying may not belong to the equilibrium. This outcome would occur when the leader applies for ATI and then triggers a price war with competitors, leading to lower expected profits compared to the case where no one is allied. We also saw that asymmetric equilibria might arise because of the heterogeneity of carriers with respect to efficiency gains. In such a setting, alliances might offer lower prices.
However, brand loyalty ensures demand for non-allied companies even if they offer higher prices.

Welfare analysis suggests that if the efficiency gains are high enough, carriers may be interested in reaching an agreement, increasing total welfare. However, in some cases the leader decides to coordinate even if the alliance creates negative synergies, with the objective of deterring the alliance formation by the follower. In this case, competition authorities should be concerned about granting ATI to the leader.

Our simple model can accommodate interesting extensions. For instance, further research could study the incentives of airlines to form alliances as a function of the number of spokes in the domestic market relative to the network of partners in the foreign country. We expect that the equilibrium behavior of carriers depends on how important the international demand is with respect to the domestic one. The model can also be extended introducing other decision variables as frequency of flights.

REFERENCES


Figure 1. Net Structure

Figure 2. Timing

Figure 3. Airlines 1 and 3 Non-Allied: Airline 2 Choice
Figure 4. Airlines 1 and 3 Allied: Airline 2 Choice

Figure 5. Best Response of Airline 2: Non-Alliance

Figure 6. Best Response of Airline 2: Follow Carrier 1
Figure 7. Best Response of Airline 2: Alliance

Figure 8. Best Response of Airline 2: Opposite Reaction

Figure 9. Best Response of Airline 2
7. Appendix 1

This appendix shows the necessary and sufficient conditions for parameters $\delta$ and $\theta$ to ensure strict concavity of the profit functions with respect to fares when airlines choose different strategies.

A function is strictly convex if the principal minors of its Hessian matrix are strictly positive. In our case, we want the profit function being strictly concave. For that reason, we multiply our profit functions by $-1$.

Assume that carrier 1 decides to ally and carrier 2 does not (similar results hold the other way around). Then, the Hessian matrix for minus the profit function of carrier 1 is

$$H(-\Pi_{a1n2}^{(1)}) = \begin{pmatrix}
\frac{4}{\delta} - \frac{8\theta}{\delta^2} & -\frac{4\theta}{\delta^2} & -\frac{4\theta}{\delta^2} \\
-\frac{4\theta}{\delta^2} & \frac{2}{\delta} - \frac{2\theta}{\delta^2} & -\frac{2\theta}{\delta^2} \\
-\frac{4\theta}{\delta^2} & -\frac{2\theta}{\delta^2} & \frac{4}{\delta} - \frac{2\theta}{\delta^2}
\end{pmatrix}$$

where the first column relates to fares for international markets ($XY$), the second for the city pair ($AB$), and the last one for non-stop domestic markets ($XH$).

Let $D_k^1$ denote the determinant of the principal minor of the Hessian. The corresponding principal minors are strictly positive if and only if

$$D_1^1 = \frac{4}{\delta} - \frac{8\theta}{\delta^2} \rightarrow \delta > 2\theta > 0$$
$$D_1^2 = \frac{8}{\delta^2} - \frac{24\theta}{\delta^3} \rightarrow \delta > 3\theta > 0$$
$$D_1^3 = \frac{32}{\delta^3} - \frac{112\theta}{\delta^4} \rightarrow \delta > \frac{7}{2}\theta > 0$$

We need that the three conditions hold. Hence, $\delta$ must be strictly greater than $\frac{7}{2}\theta$, since it is the most restrictive condition.

Similarly, the Hessian matrix for minus the profit function of carrier 2 is

$$H(-\Pi_{a2n1a2}^{(2)}) = \begin{pmatrix}
\frac{8}{\delta} - \frac{8\theta}{\delta^2} & -\frac{4\theta}{\delta^2} & -\frac{4\theta}{\delta^2} \\
-\frac{4\theta}{\delta^2} & \frac{2}{\delta} - \frac{2\theta}{\delta^2} & -\frac{2\theta}{\delta^2} \\
-\frac{4\theta}{\delta^2} & -\frac{2\theta}{\delta^2} & \frac{4}{\delta} - \frac{2\theta}{\delta^2}
\end{pmatrix}$$

Let $D_k^2$ denote the determinant of the principal minor of the Hessian. The corresponding principal minors are strictly positive if and only if

$$D_2^1 = \frac{8}{\delta} - \frac{8\theta}{\delta^2} \rightarrow \delta > \theta > 0$$
$$D_2^2 = \frac{16}{\delta^2} - \frac{32\theta}{\delta^3} \rightarrow \delta > 2\theta > 0$$
$$D_2^3 = \frac{64}{\delta^3} - \frac{160\theta}{\delta^4} \rightarrow \delta > \frac{5}{2}\theta > 0$$

We again need that the three conditions hold. Hence, $\delta$ must be strictly greater than $\frac{5}{2}\theta$, since it is the most restrictive condition.

We want that both functions are strictly convex to ensure strict concavity of the profit functions. Then necessarily,
\[ \delta > \frac{7}{2} \theta > 0 \]

For simplicity in computations we are still more restrictive by imposing

\[ \delta > 4 \theta > 0 \]

Finally, to ensure non-negative prices we assume \( \delta \leq 1 \). Hence,

\[ 1 \geq \delta > 4 \theta > 0 \]